NUMERICAL SIMULATION OF STRONG DISCONTINUITIES IN A CONDUCTING PLASMA

THESIS SUBMITTED

BY

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C E R T I F I C A T E

This is to certify that the work embodied in the thesis entitled "NUMERICAL SIMULATION OF STRONG DISCONTINUITIES IN CONDUCTING PLASMA" being submitted by KISHOR KUMAR SRIVASTAVA , M.Sc.for the award of the degree of Doctor of Philosophy of the Bundelkhand University Jhansi , has been carried out under my supervision and guidance , that the work embodied has not been submitted elsewhere for the award of any other degree and is up to the mark both in it's academic contents and the quality of presentation .

It is also to certified that Mr. Kishor Kumar Srivastava has put up attendence of more than 200 days in the department.

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A C K N O W L E D G E M E N T S

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P R E F A C E

The present thesis an out come of researches carried out by me in the field of "NUMERICAL SIMULATION OF STRONG DISCONTINUITIES IN A CONDUCTING PLASMA" under the supervision of Dr V.K. Singh ,Ph.D in the department of Mathematics & Statistics Bundelkhand University Jhansi (presently Asstt. Professore,in the department of Maths, Longowal Institute of Engg . and Technology Longowal,Sangrur), is bieng submitted for the award of Ph.D . degree in Mathematics .The thesis has been devided into eaight chapters , each chapter has been subdeivided into a number of articles .

The first chapter is introductory . It gives inbrief ,an idea about shock waves, spherical and cylindrical shock wave , equations of motion and jump condition behind the shocks in magnetogasdynamics ,radiation phenomenon , similarity Principle, characteristic method ,artificial viscosity and Withom's rule .

The second chapter is devoted to unify study of magnetoradiative shock wave propagation in conducting plasma , taking line explosion in a gas cloud and a detail study is made for the propagation of spherical and cylindrical shock wave . Numerical integration of differential equations of motion is done on D.E.C 1090 computer at I.I.T Kanpur by well Khnown R.K.G.S Programme.

Chapter third deals with self similar magnetogasdynamic cylindrical shock waves with transverse and azimuthal axial magnetic field . The motion of gas is assumed to be adiabatic and

total energy of the wave remain cosntant . In this chapter instataneous release of energy along a line in a gas cloud is assumed . A comparative study is made between cylindrical shock wave with transverse and axial magnetic field. Numerical integration of differential equation's is carried out on DEC 1090 computer system at I.I.T Kanpur by R.K.G.S progromme .

In fourth chapter , an attempt is made for the study of spherically symmetrical strong discontinuities with increasing energy in generalized Roche model and numerical solution are also obtained when the radiation heat flux is more imoortant then the radiation pressure and radiation energy . The effect of magnetic field has also been taken in to accout .

Fifth chapter consists of analysis of self similar motion in the theory of stellar explosion, taking Newtonian gravitation into account. A thorough analysis of self similar equations of gas dynamics, under the effect of magnetic field is developed. A number of new solutions has been obtained in which radial oscillation of gas occur after the shock wave passes .It is supposed that star is in equilibrium state .

Sixth chapter deals with the study of an exact solutions of normal shock wave of varibale strength advancing into a region of variable density taking pressure as constant and density varies according to as power law .

In seventh chapter,an attempt is made for the analytical solution of cylindrical shock wave in a rotating gas with azimuthal magnetic field ,taking density as constant ahead the

shock front. By using C.C.W method,analytical solutions are have been obtained for shock velocity and shock strength for weak and strong magnetic field.For strong shock ,also we have considered two cases ie.when the magnetic field is strong and when weak, ie. non magnetic case.

Eaighth chapter consist , the effect of artificial viscosity on the explosion of discontinuities in a rotating interplanetry medium, an analytical solution where a very thin velocity gradiant exists has been obtained. The concept of artificial viscosity suggested by Ritchmyer - Von Neumann has been introduced. It is assumed that velocity gradient is function of coordinates only. The effect of coriolis force has been considered due to circular rotation of medium in the space. Expressions for two different cases, the weak shock strength and the strong shock strength are obtained.

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C H A P T E R - I

I N T R O D U C T I O N

(1) SHOCK WAVES :-

When disturbances of finite amplitude are propagated in perfect fluid or gases that is those with no viscosity or heat conductivity , discontinuities in pressure and velocity to the medium may occur . These are called shock waves or shock . The reason for their development may be seen in the case of one dimensional flow such as occurs in a tube of uniform cross section when a disturbance caused by the motion of a piston at one end is propagated down the tube. The principles of conservation of mass, momentum and energy is still apply across these plane discontinuities. These disturbances propagated through out the fluid as a wave motion and with the speed of sound relative to the fluid without suffering any destoration. It may be pointed out that shock wave is a surface of discontinuity , pulse like in nature and is sometimes more appropriately called shock front ([1],[2],[3],[4]).In general discontinuity surfaces are of two type - contact surface and shock front . A contact surface is a surface separating two parts of the medium with out any flow of a gas through the surface . A shock front is a discontinuity which is crossed by gas.

The velocity [u] of the piston at any time is communicated to the gas and propagates down the gas with velocity of sound [c] relative to the gas. Hence relative to the tube the velocity is propagated with the speed (c + u) in the direction away

from the piston. Now if the piston moves to compress the gas in the tube that is , to decrease the volume of the gas, then the velocity of the gas inereases since the temperature and density increases. Thus the greater the velocity, the greater the speed with which it is propagated if this process continued indefinitely,lesser velocities would be over taken by greater ones and we would have two values of the velocity at a given place in the gas which is impossible.

However, before this occurs there will be a time at which the velocity profile of the gas, that is the velocity distance curve, has a vertical shape. In this case, the differential equation governing the motion break down and the basis for the statement that the fluid velocity (c + u), is no longer true .

Before formulating these laws for a perfect fluid it may be pertinent to point out that it is consequence of the non - linear character of the equations governing the propagtion of finite disturbance ; that is in part responsible for the prediction of the formation of the discontinuities. Thus in the example mentioned above it is essential to the argument that the local velocity of sound (c) be greater at points where the velocity of the gas is larger than velocity of sound. This is the case for a compressive motion of the piston because the disturbance which have passed over the gas have changed its character(when heated it) and hence subsequent disturbance are travelling in a defferent medium than the original one. Taking in to account the change in the medium produced by one part of the phenomenon in the discussion above , subsequent part is accomplished mathematically by the non linear terms in the equation of motion.

The non - linear character alone is not enough to cause discontinuities for it,in the example given above,the piston motion were such as to increase the volume of the gas in the tube. that is,if a rarefraction wave propagated down the tube then the local velocity of sound would decrease with increasing fluid velocity and even if a discontinuity were originally present it would disappear with time. The fact that only compression shocks are found in media which behave approximately like ideal gases is in agreement with the second law of thermodynamics :(Walker and Taub [5])

It is true of course. that a shock wave is not a discontinuity in the strict sense. It has a finite thickness across which the physical properties change continuously.If this thickness is small compared with some appropriate macroscopic dimension of the flow field. such as the radius of the curvature of a curved shock,the physical relationship may be obtained by an analysis which treats the discontinuity as strict.The assumption that the discontinuity thickness is small is a fundamental one.The term "structure" as applied to a shock wave, refers to the value of physical properties of the fluid within the small but finite thickness of the discontinuity [6].If the thermodynamic equilibrium in a substance is disturbed. a characteristic time must elapse before equiliburium can be approximately reestablished. If the physical and chemical changes occuring in the discontinuity are sufficiently slow,so that the thickness of the discontinuity is large compared with the characteristic distance,the concept of the thermodynamic quasi - equilibrium may be considered . In this case Navier - Stoke's equations are applicable .If thickness is thin , with physical and chemical

changes occurring rapidly , the essential absence of thermodynamic equilibrium must be taken into account .The first notable concept of shock waves goes almost a century back to Riemann [7],who was the first to recognise the essential difference between infinitely small and finite pressure variations .Theory of shock was later developed by Rankine [8] and Hugniot [9] and many other scholars .In fact shock wave mav cause sudden change in aerodynamic behaviour of high speed aircrafts their balance and stability .The problem of shock wave has a bearing on many problems other than those of supersonic aeronoutics also . for example , detonation waves.

The concept of magnetohydrodynamic was first introduced by Hoffmann and Taylor [10] ,taking an account of gas dynamic of steller bodies .These magnetohydrodynamic shocks are of great importance in various astrophysical ,geophysical problems and in technological developments for examples , propagation of shocks in steller atmosphere , and discontinuities in magnetoplasma of solarwind , since at very high temperature gases are ionized with high degree of ionization , a mixture of ions and electrons which is called plasma . Since most of the matter in universe is either sufficiently hot or sufficiently diffused that is in the state of plasma. We also know about the presence of cosmic megnetic field, then there is a interaction between hydrodynamic motion field which may result in magnetohydrodynamic shock is of considerable importance not only in various astrophysical and geophysical problems but also in the study of behaviour of interplanetry plasma as well as intersteller masses.

Due to the high temperature there is of interest to consider the effects of thermal radiation in gasdynamic. Sach [11] was first person who derived shock conditions taking into account radiation pressure and radiation energy.Later, Guess and Sen [12], considered the effect of radiative transfer. Marshak [13] took into account radiation flux also and he rederived the Hugnoit shock conditions and similarity solutions . Elliot [14] and Wang [15] considered the similarity solutions of Taylor's expansion problem and Piston problem taking an account of Radiation flux. Bhatnagar and Sachdev [16] studied of the propagation of isothermal shock,they used the Witham's rule to study the variations in the shock strength and shock velocity, after using pressure energy and flux, of radiation. The purpose of present work is to make thorough study of such radiative magneto-hydrodynamic shocks by formulating suitable defferential equations and reducing them to the form as to make them amenable for numerical integration.

2. SPHERICAL AND CYLINDRICAL SHOCK

Consider the propagation of a shock wave,through a perfect gas,of great intensity resulting from a strong explosion,ie from the instantaneous release of a large quantity of energy.

When the energy is suddenly released,in an infinitely concentrated form and distribution of density pressure etc, depends only on the distance from some point then this is a case of spherical shock.

When the energy is suddenly released along a line and distribution of all quantities is homogeneous in some direction and has complete axial symmetry about that direction,then this is

a case of cylindrical shock.

3. EQUATIONS OF MOTION AND JUMP CONDITION IN MAGNETO HYDRODYNAMICS

In an electrically conducting fluids in the presence of magnetic field discontinuity,ie. Shock wave in flow variable can exist's. The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of F.de Hoffmann and Taylor [10] .

When electric current induced in the fluid,then their flow in the magnetic field produces mechanical forces which modify the motion [17], magnetogas dynamic owes its peculiar interest and difficulty to this interaction between the field and the motion. Thus the equations of magnetogasdynamic are the ordinary and electromagnetic equations . We only take into account the interaction between the motion and the magnetic field ,we have ignored the Maxwell's displacement currents. Through out the thesis we take(μ)= 1 because magnetic permeability (μ) differs only slightly from unity which is unimportant. We also assume that the dissipative mechanism such as viscosity,thermal conductivity and electrical resistant are absent.

Since the problems dealt in this thesis relate to magneto gas dynamic or magneto-radiative shocks . We refer in this article the relevant flow and fluid equation [18]

$$P=\rho RT \;; \qquad (1.1)$$

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial r} + j \frac{\rho u}{r} = 0 \;; \qquad (1.2)$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\nu h^{2}}{\rho r} = 0 \; ; \qquad (1.3)$$

$$\frac{Dh}{Dt} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{\nu h u}{r} = 0 \; ; \qquad (1.4)$$

$$\frac{\partial}{\partial t}(P\rho^{-\tau}) + u\frac{\partial}{\partial r}(P\rho^{-\tau}) = 0 \; ; \qquad (1.5)$$

where $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + u\dfrac{\partial}{\partial r}$

and u, P, ρ, h, r and t are the velocity , pressure , density magneteic filed transverse to the flow , radial distance and time respectively , τ is the ratio of specific heat at constant volume and constant pressure . J=0, 1 and 2 corresponding to plane , cylindrical and spherieal respectively and V =0 for plane and V =1 corresponds to cylindrical and spherical both .

In the presence of magnetic field, the relation connecting the flow and field quantities on the two sides of the shock surface are as follows [19,20], where the velocity in front the shock wave is zero :

$$h_2 (u-u_2) = h_1 u \; ;$$

$$\rho_2 (u-u_2) = \rho_1 u \; ;$$

$$p_2 + \frac{1}{2} h_2^2 + \rho_2 (u-u_2)^2 = p_1 + \frac{1}{2} h_1^2 + \rho_1 u^2 ;$$

$$\frac{1}{2}(u-u_2)^2 + \frac{\tau p_2}{(\tau-1)\rho_2} + \frac{h_2^2}{\rho_2} = \frac{1}{2} u^2 + \frac{\tau p_1}{(\tau-1)\rho_1} + \frac{h_1^2}{\rho_1} :$$

where sufix 1 and 2 correspond to the value of the quantities just ahead and just behind the shock surface and u is the shock velocity.

(4) RADIATION PHENOMENON

At very high temperature,the gases become ionized and radiation can be considered as a continuous emission of energy in the form of electromagnetic wave which propagated in the medium with the speed of light.This energy is called radiant or thermal energy radiation . Where as according to quantum theory the radiant energy emitted or absorbed is not continuous , permitting all possible values , as demanded by the wave theroy , but in a discrete quantified form , as integral multiples of an elementry quantum of energy , photon or light quanta .The amount of energy in each quanta being given by the product $h\nu$, where h is plank's constant and ν is the frequency of the radiation.

Thus the quantum theory proposes the particles caracteristic of radiation ,while classicial theory the wave charactristics , both being required to understand the complex behaviour of

radiation . A complete study of such a high temperature flow of a gas should consist of the study of gas dynamic field , the electromagnetic field and thermal radiation flux simultaneously . The anlysis of simultaneously effects and magnetic field forms the subject matter of radiation magnetohydrodynamics . This theory of thermal radiation can be applied to understand the processes which take place in stellar media , to explain the observed luminosity of stars and nuclear explosions and also to high temperature flow.

Radiative transfer and radiative heat exchange have an influence on both the star and the motion of the fluid .This influence is caused by the fact that fluid loses or gains energy by emitting or obsorbing heat . On the flow field of the gas there are three radiation effects expressed in terms of radiation pressure , radiation energy and radiation flux [21 ,22 ,23] .

(a) RADIANT PRESSURE

By the theory of electrodynamic the pressure of a radiation field is equal to one third of radiant energy is

$$P_R = \frac{1}{3} E_r = \frac{1}{3} a_R T^4 ;$$

where T is temperature and a_R is Stefan - Boltz mann constant . This is the only component of the radiation which differs from zero .

(b) RADIATION ENERGY (E_r)

The radiant energy density E_r per unit mass of the fluid is

given by

$$E_R = \frac{a T_R^4}{\rho} ;$$

where ρ is density of fluid

(c) RADIATION FLUX

The net amount of radiant energy passing through the surface per unit area per unit time is called the radiant flux through the surface is gives by

$$F_R = \frac{C_R}{4} \cdot \frac{a T_R^4}{\rho} ;$$

or $f_R = D_R \text{ grad } E_R$

where $D_R = \frac{CL_R}{3}$ is the Rosseland diffusion coefficient of

radiation and $L_R = \frac{1}{K\rho}$ the Rosseland mean free path of radiation

Magnetic pressure P_h and the magnetic energy E_h are given by

$$P_h = \frac{h^2}{2P} ;$$

$$E_h = \frac{h^2}{2p} \quad ;$$

where h is the magnetic field

The fundamental equation of radiation magnetogasdynamics for one dimensional flow are [24]

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial r} + j \frac{\rho u}{r} = 0 \quad ;$$

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{\nu h^2}{\rho r} = 0 \quad ;$$

$$\frac{Dh}{Dt} + u \frac{\partial u}{\partial r} + \frac{\nu h u}{r} = 0 \quad ;$$

$$\frac{DE}{Dt} + P \frac{D}{Dt} \left(\frac{1}{\rho} \right) + \frac{1}{\rho r^J} \frac{\partial}{\partial r} \left(F r^J \right) = 0 \quad ;$$

$$\text{where} \quad \frac{D}{Dt} = \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) \quad ;$$

$$\text{where} \quad P = P_m + P_r + P_h \quad ;$$

$$E = E_m + E_R + E_h \ ;$$

$$E_m = \frac{P_m}{\tau - 1} \ ;$$

and F is the radiation heat Flux.

where suffixes m, R, h attachad to the quantities to expressions for material radiation and magnetic terms respectively .

5. SIMILARITY PRINCIPLE

The fluid is said to be one-dimensional when all its properties depend on only one geometric coordinate and on the time. The spherical, cylindrical and plane waves produce one dimensional motion. The methods of dimensional analysis can be used to find exact solutions of certain problems of one dimensional unsteady motion of a compressible fluid. In the Eculerian approach the basic physical variables are the velocity,u the density ρ and pressure P. The characteristic parameters are the linear coordinate [r],the time t and the constants that enter into the equations,the boundary and the initial conditions of the problem. Since the dimensions of the quantities ρ and P contain the mass,at least one constant a, the dimension of which also contain the mass must be a characteristic parameter. Hence,as in [24] we can assume with out any loss of generality

$$\dim \ 'a' = ML^k T^s \ ;$$

We can write the velocity, density and pressure as

$$U = \frac{r}{t} V, \quad \rho = \frac{a}{r^{k+3} t^{s}} G, \quad p = \frac{a}{r^{k+1} t^{s+2}} P \;;$$

where V, G and P are arbitrary quantities and, therefore, can depend only on one-dimensional combination of r, t and other parameter of the problems. In general, they are functions of two non-dimensional variables. But if an additional characteristic parameter 'b' can be introduced with dimensions independent of those of 'a'. The number of independent variables which can be formed by combining 'a' and 'b' is reduced to one. Since the dimension of the constant 'a' contains the mass, we can choose the constant 'b' in a manner, such that its dimension do not contain the mass, that is

$$\dim b = L^{m} T^{n} \;;$$

The single non-dimensional independent variable in this case will be $\dfrac{r^{m} t^{n}}{b}$, which can be replaced by m≠0 by the variable

$$\eta = \frac{r}{b^{1/m} t^{\sigma}} \,,$$

where $\sigma = - \dfrac{n}{m}$;

If $m=0$, V, G and p depend only on the time and the velocity u is proportional to r.

The solutions depending on the independent variables may obtain a number of arbitrary constants .

The above argument shows that when the characteristic parameters include two constant with the independent dimentions in addition to r and t , the partial defferieantial equation satisfied bythe velocity,density and pressure in one dimension unstedy motion as a compressible fluid can be replaced by ordinary differential equations for V,G and P. The solutions of these ordinary differential equations can some times be obtained exactly in closed form [25].For violent spherical explosions in a uniform atmosphere at rest , and in other cases,approximated by using numerical integration [26] such motion are called self similar [27].The idealized problems of a strong explosion in a homogeneous atmosphere represents a typical example of self-similar flow,in which the flow variables changes with times in such a manner that their distributions with respect to coordinate variable always remain similar in time. The self similar problem of a strong explosion was formulated and solved by Sedov [28] using a brilliant method, which employed the energy integral. Sedov, succeded in finding an exact analytic solution to the equation of self-similiar motion . The same problem was also considered by Taylor[26] who formulated the equation for the problem obtained numerical but not analytic solutions.

(9) CHARACTERISTIC METHOD

Many physical problems lead to the formulation of a quasi-linear system of first order equations,such equations are

linear in first derivatives of the dependent variables. but the coefficient may be functions of the dependent variable when these equations describe wave motion, a good understanding of many of the issues can be developed from the study of plane waves. Accordingly, we start with the case of two independent variables The two variables often the time ond space variable so we denote them by t and x ,and use corresponding terminology , but the discussion applies to any variable system if the dependent variables are U_i (x,t) i=1,2---- n, the general quasi-linear first order system is

$$A_{ij} \frac{\partial u_j}{\partial t} + a_{ij} \frac{\partial u_j}{\partial x} + b_i = 0 \quad , \qquad (1.1)$$

where the matrices A,a and vector b may be functions of v_j -- v_n as well as x and t.

In general, any one of the equations has different combination of $\partial u_i/\partial t$ and $\partial u_j/\partial x$ for u_j. That is it couples information about the rate of change of the different u_j in different directions .

One can not deduce information about the increments of all the u_j for a step in any single direction but we are at liberty to manipulate the n equal to see whether this information can be obtained from some combination of them we therefore consider the linear combination

$$l_i\left(A_{ij}\frac{u_j}{t} + a_{ij}\frac{u_j}{x}\right) + l_i b_i = 0 \quad , \tag{1.2}$$

where the vector l is a function of x,t,u and investigate whetner l can be chosen so that (1.2) takes the form

$$m_j\left(\beta\frac{\partial u_j}{\partial t} + \frac{\partial u_j}{\partial x}\right) + l b_j = 0 \quad , \tag{1.3}$$

If this is possible (1.3) provides a relation between the directional derivative of all the u_j in the single direction (α,β). When this is the case, it will be valuable to introduce curves in the (x,t) plane defined by the vector field (α,β) . If x= x(n) , t = T (n) in the parametric representation, of a typical member of this family , the total derivation of u_j on the curve is

$$\frac{du_j}{dn} = T'\frac{\partial u_j}{\partial t} + x'\frac{\partial u_j}{x} \quad ,$$

without loss of generality , we may take

$$\alpha = x'(n) \;, \beta = T'(n)$$

and writ (1.3) as

$$m_j \frac{du_j}{dn} + l_j b_j = 0 \quad . \tag{1.4}$$

The condition for (1.2) to be in the form (1.4) are

$$l_i A_{ij} = m_j T', \quad l_i a_{ij} = m_j x'$$

and we may eliminate the m_j to give

$$l_i [A_{ij} x' - a'_{ij} T'] = 0 , \tag{1.5}$$

There are n equation for the multiplyers l_i and the direction (x', T'). Since they are homogeneous in the l_i, a necessary and sufficient condition for a nontrivial solution is that the determinat .

$$\left| A_{ij} x' - a_{ij} T' \right| = 0 , \tag{1.6}$$

This is a condition on the direction of the curve such a curve in said to be a characteristic and the corresponding equation (1.4) in said to be in characteristic form .

(10) ARTIFICIAL VISCOSITY

The propagation of strong shock waves in half space due to a surface explosion or impact.has been treated in many leveles of approximation . In one of these an attempt in made to account for the material strength by including a New-tonion viscosity term. This approximation seems to have been originated by S.W vuan and various co-workers [29]. Who have further approximated the solution of the resulting equation by seeking a variety of quasi - similar solutions . In all of these solutions the viscoity coefficient is taken to be at most a function of time but independent of space co - ordinates . In addition , the flow field in the half space is represented as through it were one half of spherically symmeric flow . The predictions of this theory for certain quantities on the symmetry axis at the shock front are then composed with experiment and it is usually claimed that the agreement constitutes a validation of the model used . However it has been pointed out repeatedly that quantities at the shocks front are very insensitive to detales of model and can be aproximated fairly well by any numbers of approximations . A recent illustration of this fact is presented in a paper by Billingsley [30] whose result shows that experimental data was incapable of resolving the difference between a wide variety of theoritical treatments even in the subset of quasi-similar solutions with out viscosity . These are many remaining questions about the effects of the spherical flow assumption. The neglect of non-similarities due to non-zero projectile size , and the

appproximation used in defining the equation of state to mention only a few. These hav been critically reviewed in a recent survey paper by Rae [31] when a spherically symmetric magnetic field is added to this formulation along with a Newtonian viscosity term . The above paper proposes to study the composite effect of all the approximation .

Von-Neumann and Richtmyer [32] developed a new technique of artificial viscosity and introduced an additional term in the following form

$$q = \frac{1}{2} (\rho_0 C \Delta x^2 (\left| \frac{\partial v}{\partial t} \right| - \frac{\partial v}{\partial t})) ,$$

where C is a dimensionless Constant .V the material velocity x the internal length and ρ_0 the density at t=0. We have shown in this thesis that artificial viscosity can smear out shock discontinuity in a magnefied medium more easily than in ordinary gas dynamics.

(ii) WITHAM'S RULE

Chester [33] has studied the motion of a shock wave down a non uniform tube on the basis of a linearlized theory in which the changes in the tube area and the consequent changes in shock strength have been assumed to be small. In this linearlized theory,the solutions breaks down when the flow behind the shock wave is nearly sonic Chisnel [34] and Witham [20] have shown that Chesters work could be simplified and extended one minor simplification is that where as the Chester worked with the full

three dimensional equations and performed and averaging process in the course of his analysis. It is sufficient to work from the outset with the one diamensional formulation . Witham has shown that the motion of the shock can be found in a simple way without solving the equations for the flow behind the shock in detail . This method can be givin as the following rule .

The relevant equations of motions are first written in characteristic form . Then the rule is to apply the differential relation which must be satisfied by the flow quantities along a characteristic to the flow quantities just behind the shock wave . Together with the shock relations which express these values in terms of the shock strength , the rule determines the change in the shock strength . In fact it is found to the accurate even when the total changes in the shock strength are not small although a full understanding of this fact is still lacking .

R E F E R E N C E S

[1] F. Chorlton : Textbook of fluid dynamics : D. Van Nostrand IDT London (1967).

[2] R.Courant : Supersonic flow and shock waves
And
K.D.Freidriche: Inter sceince publication INC. New York (1940)

[3] W.Bleakney : Interaction of shock waves : Rev. Mod
and
A.H.Taub phys :21 ,58 (1949) .

[4] S.I. Pai : Introduction to the theory of compressible flow :D Van Nostrand company INC. New York (1959).

[5] Walker and Taub: Reviewed modern physics : vol number 4 page 585. (1949)

[6] Howard W.Emmons: Fundamentals of gas dynamics :Princeton University Press , New Jersey (1958).

[7] R.P.Kanwal : On magneto hydrodynamic shock waves; J. Math.Mech; 9,681 (1960).

[8] W.J.M.Rankine : On thermodynamic theory of waves of finite lohgitudinal disturbances.
Thansaction of royal society of London ; 160 277 (1870).

[9] H.Hugniot : Journale de iocole polytechnique ; 58,1 (1889).

[10]D.Hoffman : Magnetohydrodynamic shocks ;Phys Rew.80, 692 (1950).
and
R.Taylor

[11] R.G.Sache : Phys Rev: 69 , 514 (1946).

[12] H.K.Sen and A.W.Guess : Phys.Rev.: 108,560 (1957)

[13] R.E.Marshak : Phys fluids;1,2,(1958).

[14] L.A.Elliot :Proc Royal Soc :258 A, 287 (1960).

[15] K.C.Wang : J fluid Mech ; 20,447 (1984).

[16] P.L.Bhatnagar and P.L.Sachdev : Nuobo Cimento : 44 , 15 (1966).

[17] L.D.Landau and E.M.Lifshdtz : Electrodynamics of continuous media : Pergamon Press ; New york (1960).

[18] T.G.Cowling : Megneto hydrodynamics : Interscience Publication Inc , New York (1957).

[19] S.I.Pai : Megnetogasdynamics and plasma dynamics . Springer Verlo (1962).

[20] G.B.Witham : J.Fluid Mech :4,337 (1958).

[21] J.D.Jackson : Clasical electrodynamics ;John Willy and sons Inc, New York. (1975)

[22] S.Chandrashekhar: An introduction to the study of stellar structure ; Dover publications Inc, New York. (1957)

[23] YA.B.Zel'dovich and YU.P.Raizer : Shock waves and radiation. An article in annual review of fluid mechanics; 1 ,385 (1959).

[24] L.I.Sedov : Similarity and dimensional methods in mechanics ; Academic press,New York (1959).

[25] C.J.Kynch : Modern developments in fluid dynamics High Speed Flow;Clarendon Oxford (1953).

[26] G.I. Tayler : Proc. Roy.Soc.London: A 201.159,(1950).

[27] G.A.Liubinov : On possible kinds of one dimensional Unsteady Viscous gas motion:A paper in thermodynamics. Vol 7.Moscow

[28] L.I.Sedov : Compt.Rend (Doklady) Acad.Sci.USSR 52.17 Prinkl. Math.1, Mech.10.241 (1946).

[29] S.W.Yuan and C.N.Scully :A new approach to hyper velocity impact theory;PP 599-615 of the procedings of the 9th annual meeting of the American astronoutical society (1963).

S.W.Yuan : An analylical approach to hyper velocity impact: AIAA Journal (1971).

J.L.Whitesider and S.W. Yuan : Viscouss effects on hyper velocity impact "Journal of applied physics 42,4158 (1971).

J.L.Whitesder and S.W.Yuan : Influence of viscosity on blast wave solutions as applied to hyper velocity impact "International journale of engineering science,10,337 (1970)

[30] J.P.Billingsley : Comparision on experimental and predicted arial pressure variation for semi-infinite metalic targets; AIAA paper 69-361

[31] W.J.Rae : "Analytical studies of impact generated shock propagation survey and new results;" R. Kinslaw , Acad. Press. (1970)

[32] J.Von Nevmann : J.appl.Phys.:21, 232 (1950).

and

R.D.Richtmyer

[33] W.Chester : Phil Mag; 45, 1243 (1954).

[34] R.F.Chisnell : J fluid Mech : 2.286 (1957).

CHAPTER - II

MAGNETO - RADIATIVE SHOCK WAVE PROPAGATION IN A CONDUCTING PLASMA

1. INTRODUCTION

The problem of propagation of shock wave in a non homogeneous medium has been studied by many authors where the effect of explosion in the stars and atmosphere of the earth are discussed .

The solutions of cylindrically symmetric flow has been obtained by Lin [1].Ray [2] has discussed the problems of point and line explosion and found an exact analytic solution . Analytic solution in the three cases of plane ,cylindrically symmetrical , Spherically symmetrical flows have also been discussed by Sakurai [3] . Rogers [4] has also studied the similarity solutions for all the three cases in uniform atmosphere .Later on Singh and Vishwakarma [5] have discussed the similarity solutions of the flows behind shock waves in a radiative magnetogasdynamics in which total energy increases with time.

In this chapter the problem of explosion along a line in a gas cloud has been discussed , similarity solutions are developed describing the propagation of a cylindrical and spherical shocks in a non - uniform atmosphere where magnetic effect is observed due to ionization of gases and the problem is solved , taking counter gas pressure and radiation heat flux in to account . The radiation pressure and radiation energy are ignored due to weak ionization of gases . The gas in the undisturbed field is assumed

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to be at rest . We also assume the gas to be grey and opaque and the shock to be transparent and isothermal. The total energy of the explosion is constant . The Runge Kutta method has been used for approximations on computer system DEC 1090 at IIT kanpur .

(2) SELF - SIMILAR FORMULATION

The fundamental equations of conservation of mass , momentum energy and equation of the magnetic field behind the wave are given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + J\,\frac{\rho u}{r} = 0 , \qquad (2.1)$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial r}(\rho u^{2}) + \frac{\partial P}{\partial r} + h\,\frac{\partial h}{\partial r} + \frac{\nu h^{2}}{r} = 0 , \qquad (2.2)$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial r}(\rho u E) + \frac{\partial}{\partial r}(P u) + \frac{1}{r^{J}}(q r^{J}) = 0 , \qquad (2.3)$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial r}(u h) + \frac{\nu h u}{r} = 0, \qquad (2.4)$$

where $J=1,2$ for cylindrical and spherical respectively. and $\nu=1$. r is the radial distance from the line of explosion, density ρ , pressure P,radial velocity u heat flux q, time internal energy E, and h is the magnetic field.

For an ideal gas internal energy E is given by

$$E = \frac{P}{(\tau-1)\rho} = \frac{\Gamma T}{(\tau-1)} \; ; \; E = E\,(T)\,, \qquad (2.5)$$

where τ is the ratio of two specific heat . Furthermore , the equation of state of the gas is taken to be of the form .

$$P = \Gamma\,\rho\,T\,, \qquad (2.6)$$

where Γ is the gas constant .

It is assumed the local thermodynamic equilibrium and taking Rosseland's diffusion approximation (Zelodvich Raizers [6]).

$$q = \frac{-c\mu}{3}\,\frac{\partial}{\partial r}\,(\sigma T^{4}),$$

where T is the absolute temperature $\sigma C / 4$ is the Stefan Boltzmann constant, C is the velocity of light and μ , the mean free path of raidiation is a function of density and absolute temperatune T .

Following Wang [7]

$$\mu = \mu_{0}\,\rho^{\alpha}\,T^{\beta}\,, \qquad (2.8)$$

where μ_{0} , α and β are constants .The disturbance is headed by an isothermal shock surface , hence the condition across it are

$$\rho_1 V = \rho_2 (V - U_2) = m_s , \tag{2.9}$$

$$P_2 - P_1 + \frac{h_2^2}{2} + \frac{h_1^2}{2} = m_s u_2 , \tag{2.10}$$

$$E_1 + \frac{P_1}{\rho_1} + \frac{1}{2} V^2 + \frac{h_1^2}{\rho_1} = E_2 + \frac{P_2}{\rho_2} + \frac{1}{2} (V - u_2)^2 + \frac{h_2^2}{\rho_1} - \frac{q_2}{m_s} \tag{2.11}$$

$$h_1 u = h_2 (v - u_2) , \tag{2.12}$$

$$T_2 = T_1 , \tag{2.13}$$

where subscripts 1 and 2 are for the regions just out side and just in side shock surface respectively . V denotes the shock velocity and m_s be the mass flux perunit area acros the shock .

(3) SOLUTIONS OF THE EQUATIONS

In front of shock in the undisturbed gaseous medium , the density , pressure and magnetic field distribution are

$$\rho_i = AR^{w} \quad , \qquad -2 < W < 0 \tag{3.1}$$

$$P_i = BR^{n} \quad , \qquad n < 0 \tag{3.2}$$

$$h_i = DR^{m} \quad , \qquad m < 0 \tag{3.3}$$

where R is the shcok radius at the time t ; A , W , B , n , D and M, are constant .

The similarity variables which reduces the equation governing the flow to ordinary differential equation is taken as

$$\eta = r^{a} t^{b} \quad , \tag{3.4}$$

where a and b are constant . It is assumed that the flow is headed by a shock front given by

$$\eta = \eta_o \quad ,$$

the other transfromation for the flow variables are

$$U = \frac{r}{t} V(\Pi) \quad , \tag{3.5}$$

$$\rho = r^{k} t^{\lambda} \Omega(\Pi) \quad , \tag{3.6}$$

$$P = r^{k+2} t^{\lambda-2} P(\Pi) \quad , \tag{3.7}$$

$$h = r^{\frac{(k+2)}{2}} t^{\frac{(\lambda-2)}{2}} H(\Pi) \quad , \tag{3.8}$$

$$q = r^{k+3} t^{\lambda-3} F(\Pi) \quad , \tag{3.9}$$

where $\Pi = r^{a} t^{b}$,

The total energy of the disturbance perunit length is

$$Q = 2\pi J \int_{0}^{R} [\frac{1}{2} \rho u^{2} + \frac{P}{(\tau - 1)} + \frac{h^{2}}{2}] r dr \quad , \tag{3.10}$$

Interms of variable Π we have

$$Q = \frac{2\pi J}{a} \int_0^{\Pi_0} \left[\frac{1}{2} \Omega(\Pi) V^2 + \frac{P}{(\tau - 1)} + H(\Pi) \right]$$

$$* \, \Pi^{(k+4)/(a-1)} \, t^{\left[\lambda - 2 - \frac{a}{b}(k+4) \right]} * d\Pi \,, \qquad (3.11)$$

where Π_0 is the value of Π at the shock front

$$\Pi = E t^{\mu} \,, \qquad 0 < \mu = 1$$

where shock surface to be given by Π_0 = constant . This fixes the velocity of the shcok surface as

$$V = \frac{\mu - b}{a} \frac{R}{t} \,, \qquad (3.12)$$

which represents an out going surface if a < 1 . The total energy of disturbance within the shock surface at any time t is constant . This by (3.11) requires that

$$\lambda - 2 - \frac{a}{b} (k+4) = 0 \,. \qquad (3.13)$$

Let the Mach and Alfven Mach number's at the shock front respectively be defined by

$$M^2 = \frac{V^2 \rho_1}{\tau p_1},$$

$$M_A^2 = \frac{V^2 \rho_1}{\tau h_1^2} . \qquad (3.14)$$

By direct substitution of (3.5) - (3.9) in the equation of motion (2.1) - (2.8) shock condition (2.9) - (2.13) and after utilising relation (3.13), we find the similarity condition, following Singh and the Vishwakarma [5] are compatable when

$$k=w , \quad \alpha=0 , \quad a= -(4+w) , \quad b=2 , \quad n= -2 ,$$

$$\text{and} \quad m = \frac{w+2}{2} , \quad \beta = \frac{-5}{2} , \quad \alpha = \frac{w+1}{w} . \qquad (3.15)$$

Hence pressure ditribution becomes

$$P_1 = BR^{-2} , \qquad (3.16)$$

and equation (2.1) - (2.4) and (2.7) are now transformed into the forms .

$$\frac{\Omega'(\eta)}{\Omega(\eta)} = \frac{(4+W)\eta V'(\eta) - (W+1+J)V(\eta)}{[2-(4+W)V(\eta)]\eta}, \tag{3.17}$$

$$\frac{P'(\eta)}{P(\eta)} = \frac{\Omega(\eta)}{p(\eta)} \frac{[\eta V'(\eta)\{2-(4+W)V(\eta)\} + V(\eta)\{V(\eta)(1-J)-1\}]}{\eta(4+W)}$$

$$+ H^2(\eta)\frac{[(2\nu - 2) - \eta[(4+W)^2 V'(\eta)]]}{\eta[4+W][2-(4+W)V(\eta)]} + \frac{(W+2)}{\eta(4+W)}, \tag{3.18}$$

$$\frac{F'(\eta)}{F(\eta)} = \frac{P(\eta)}{F(\eta)} \frac{[\tau(3+W)V(\eta) - (4+W)\tau\eta V'(\eta) - 2]}{\eta[\tau-1](4+W)},$$

$$+ \frac{P'(\eta)}{F(\eta)} \frac{[2-(4+W)\tau V(\eta)]}{(\tau-1)(4+W)} + \frac{(W+J+3)}{\eta(4+W)}, \tag{3.19}$$

$$\frac{F(\eta)}{P(\eta)} = -N\frac{(P(\eta)/A)^{1/2}}{(\Omega(\eta)/A)^{3/2-\alpha}}[2-(4+w)\eta\{\frac{P'(\eta)}{P(\eta)} - \frac{\Omega'(\eta)}{\Omega(\eta)}\}] \tag{3.20}$$

where $N = \dfrac{4\sigma C\mu_0}{3\Gamma^{3/2} A^{1-\alpha}}$ =a non dimensinoal parameter (3.21)

$$\frac{H'(\eta)}{H(\eta)} = \frac{\eta[4+W]\,V'(\eta) - (W/2 + \nu + 2)\,V(\eta) + 1}{\eta\,[2 - (4+W)\,V(\eta)]}, \tag{3.22}$$

$$V'(\eta) = \frac{1}{N}\,\frac{(F(\eta)/A)\,(\Omega(\eta)/A)^{\frac{1}{2}-\alpha}}{[P(\eta)/A]^{1/2}}\,\frac{[2 - (4+w)\,V(\eta)]}{\eta[\{2 - (4+w)\,V(\eta)\}^2 - (4+\omega)^2 (H^2(\eta)+1)\frac{P(\eta)}{\Omega(\eta)}]}$$

$$- \frac{P(\eta)}{\Omega(\eta)}\,\frac{[2W + 2(\tau-1)\,H^2(\eta) - (J+1)(4+W)\,V(\eta)]}{\eta\,[\{2 - (4+w)\,V(\eta)\}^2 - (4+\omega)^2 [H^2(\eta)+1]\frac{P(\eta)}{\Omega(\eta)}]}$$

$$- \frac{[2 - (4+W)\,V(\eta)]\,V(\eta)\,\{V(\eta)(1-J) - 1\}}{\eta\,[\{2 - (4+W)\,V(\eta)\}^2 - (4+\omega)^2 [H^2(\eta)+1]\frac{P(\eta)}{\Omega(\eta)}]}. \tag{3.23}$$

The approximate shock condition are

$$V(\eta_0) = \frac{2}{(4+w)}\left[1 - \frac{1}{\tau M^2} - \frac{1}{\tau M_A^2}\right], \tag{3.25}$$

$$\Omega(\eta_0) = \frac{\tau M^2 M_A^2}{(M^2 + M_A^2)}, \tag{3.26}$$

$$P(\eta_0) = \frac{4}{(4+W)} \; \frac{M_A^2}{(M^2 + M_A^2)} , \qquad (3.27)$$

$$F(\eta_0) = \frac{-4}{(4+W)} \left(\frac{M_A^2 + M^2}{[\tau M_A + M^4]} - 1 \right), \qquad (3.28)$$

$$H(\eta_0) = \frac{4}{(4+W)^2} \left[\frac{3}{(\tau M)^2} + \frac{2 \tau M_A^4}{\tau M^2 (M_A^2 + M^2)} \right] , \qquad (3.29)$$

when $\eta_0 = 1$

(4) RESULTS AND DISCUSSION

For exihibiting the numerical solution it is convenient to write the flow and field variables in the following non - dimensional forms

$$\frac{U}{U_2} = \frac{(\eta_0)^{1/(4+w)}}{(\eta)} \qquad \frac{V(\eta)}{V(\eta_0)} , \qquad (4.1)$$

$$\frac{\rho}{\rho_1} = \left(\frac{\eta_0}{\eta}\right)^{w/(4+w)} \frac{\Omega(\eta)}{\Omega(\eta\ )} , \tag{4.2}$$

$$\frac{P}{P_2} = \left(\frac{\eta_0}{\eta}\right)^{(w+2)/(w+4)} \frac{P(\eta)}{P(\eta_0)} , \tag{4.3}$$

$$\frac{q}{q_2} = \left(\frac{\eta_0}{\eta}\right)^{(w+3)/(w+4)} \frac{F(\eta)}{F(\eta_0)} , \tag{4.4}$$

$$\frac{h}{h_2} = \left(\frac{\eta_0}{\eta}\right)^{(w+2)/2(w+4)} \frac{H(\eta)}{H(\eta_0)} , \tag{4.5}$$

Numerical results for certain choice of parameter are repoduced in a graphical forms. Calculations are made for following value of parameters.

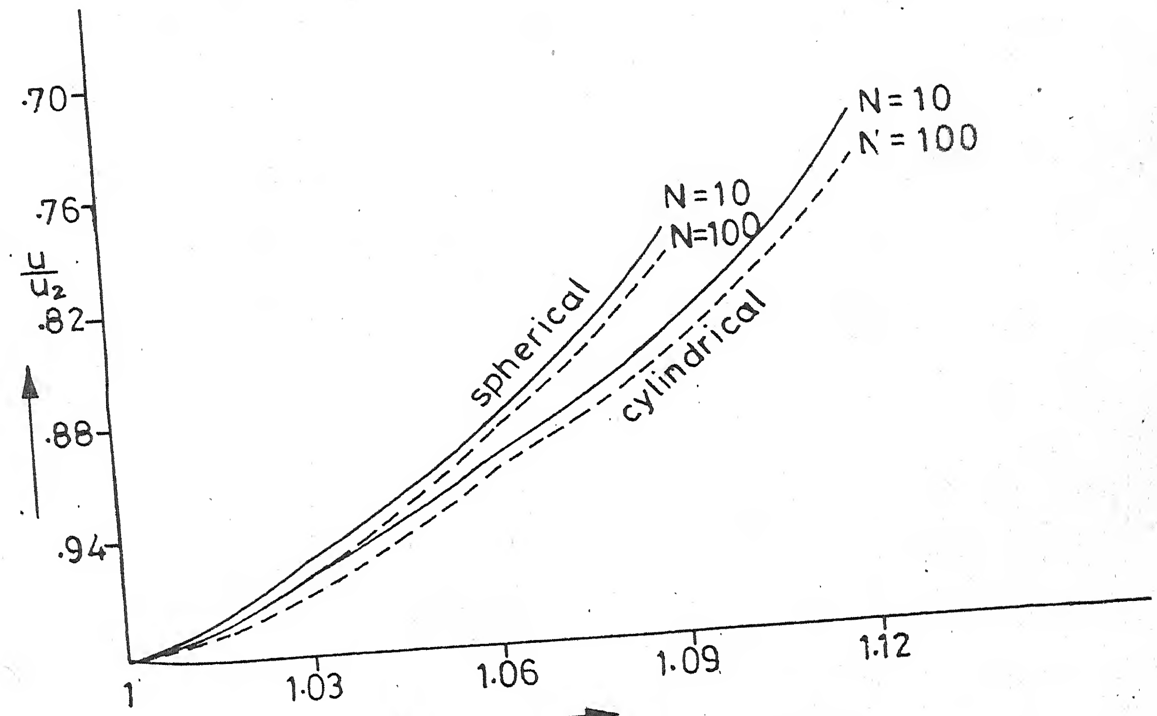
(i) $\tau=4/3$, $M^2=20$, $M_A^2=30$, $W= -1.5$, $\alpha=1/3$, $J=1$, $N=10$ $\vartheta=1$

(ii) $\tau=4/3$, $M^2=20$, $M_A^2=30$, $W = -1.5$, $\alpha=I/3$, $J=1$, $\vartheta=1$, $J=1$, $N=100$

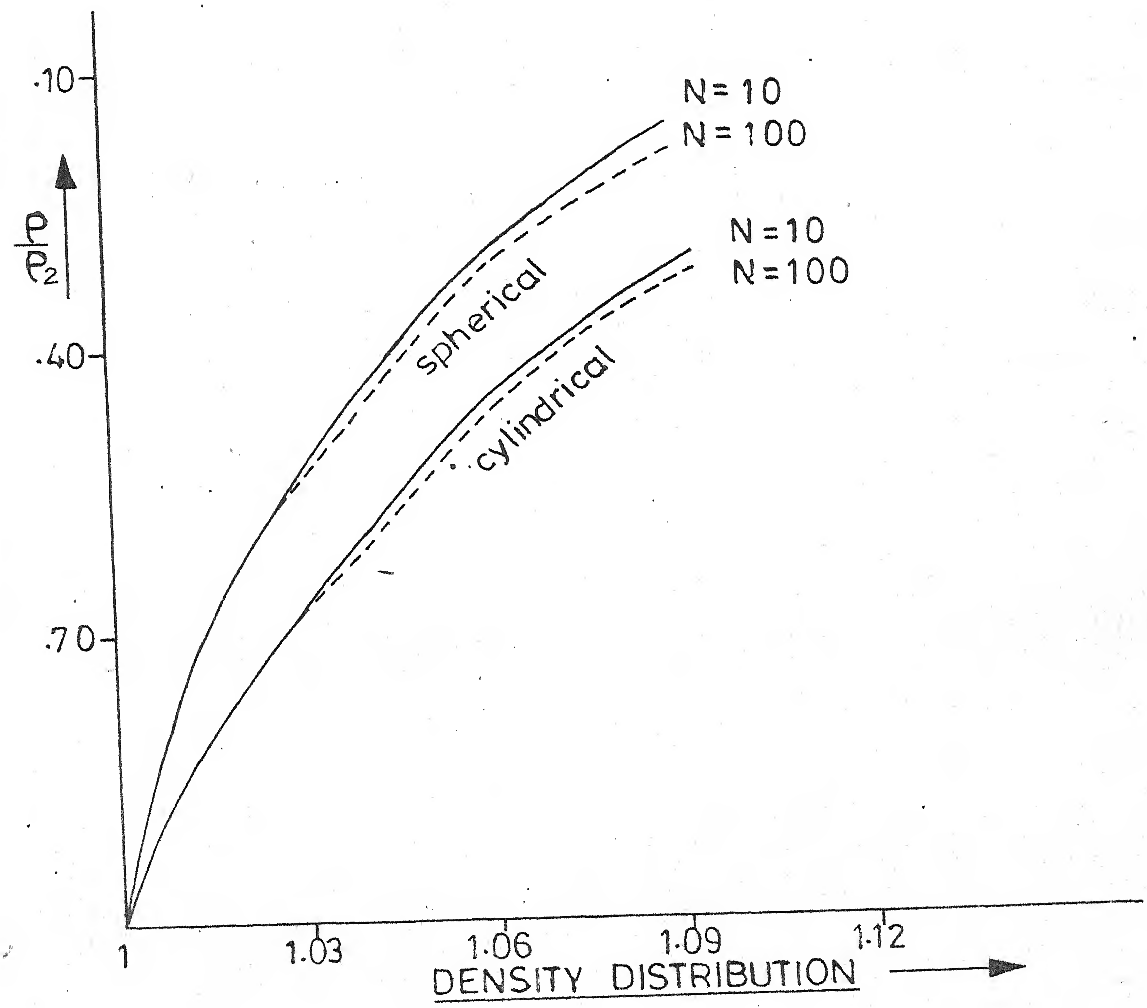
(iii) $\tau=4/3$, $M^2=20$, $M_A^2=30$, $W=-1.5$, $\alpha=1/3$, $J=2$, $\nu=1$, $N=10$

[iv] $\tau=4/3$, $M^2=20$, $M_A^2=30$, $W=-1.5$, $\alpha=1/3$, $J=2$, $\nu=1$, $N=100$

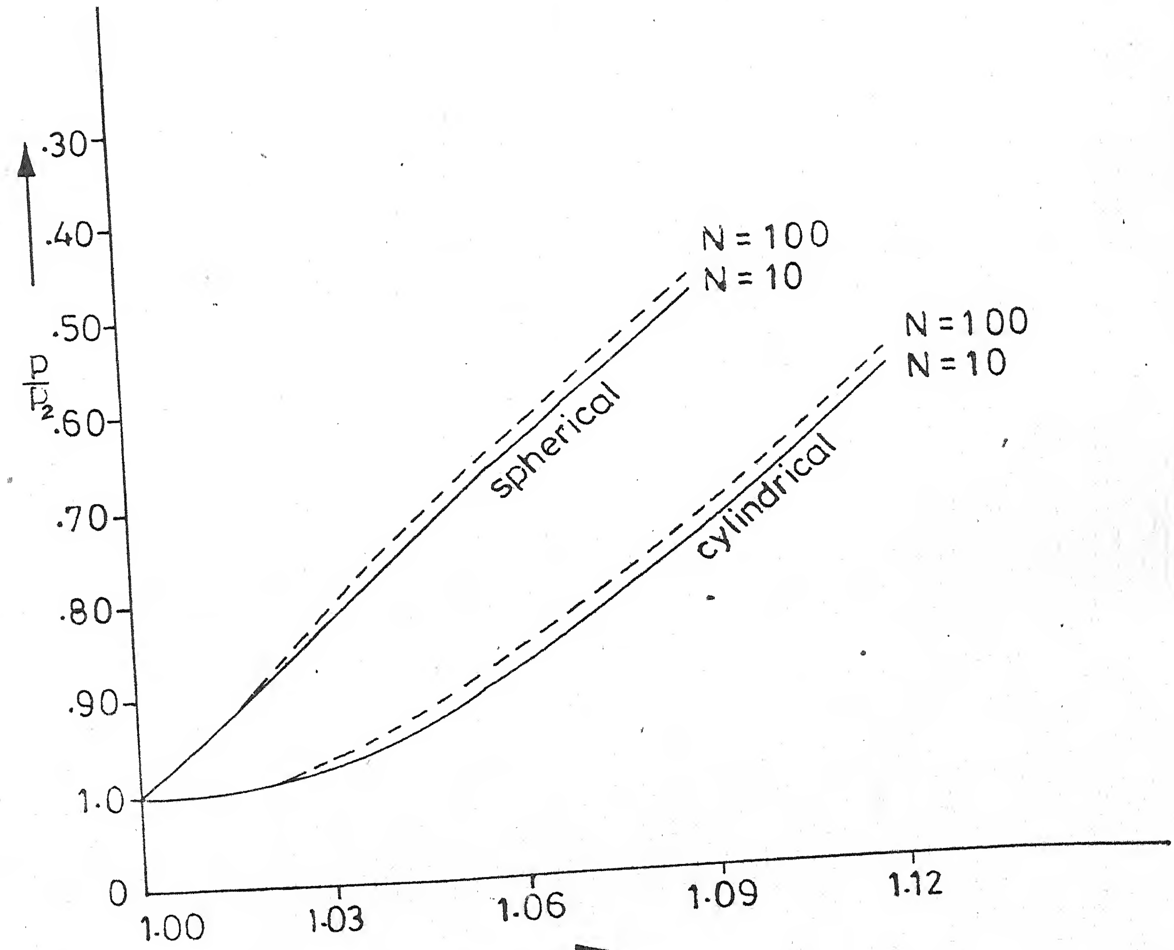
From the graphs it is clear that the velocity, density pressure,magnetic field distribution and heat flux are maximum at the shock front and decrease more rapidly towards the line of explosion as the value of Ω increases and there is also slight charge in these variables with the change of radiation parmeter. Through the graphs it is also clear that decrease in velocity , density , pressure , radiation flux and magnetic field variables is more in spherical case than decrease in cylindrical case .



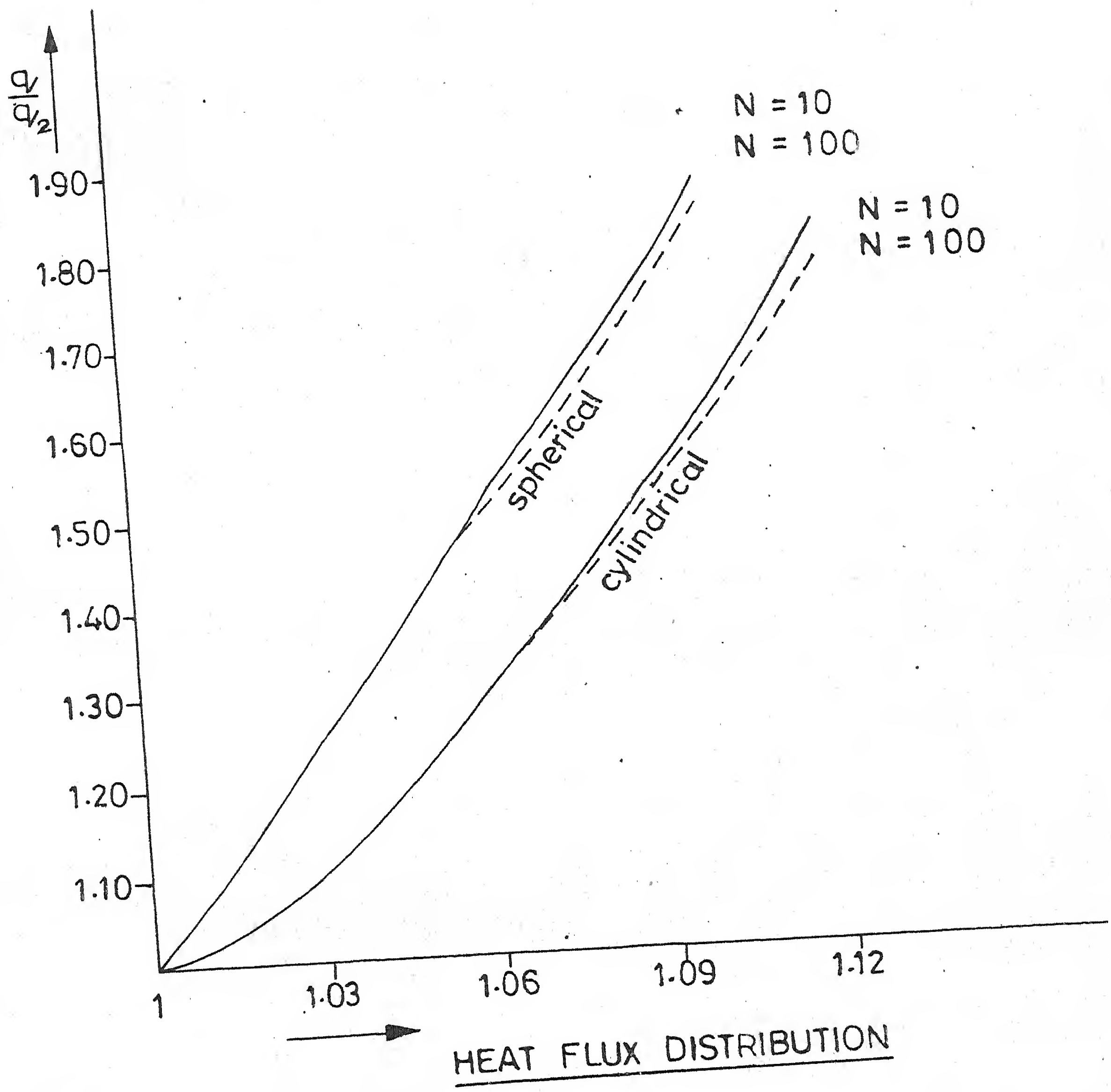
VELOCITY DISTRIBUTION



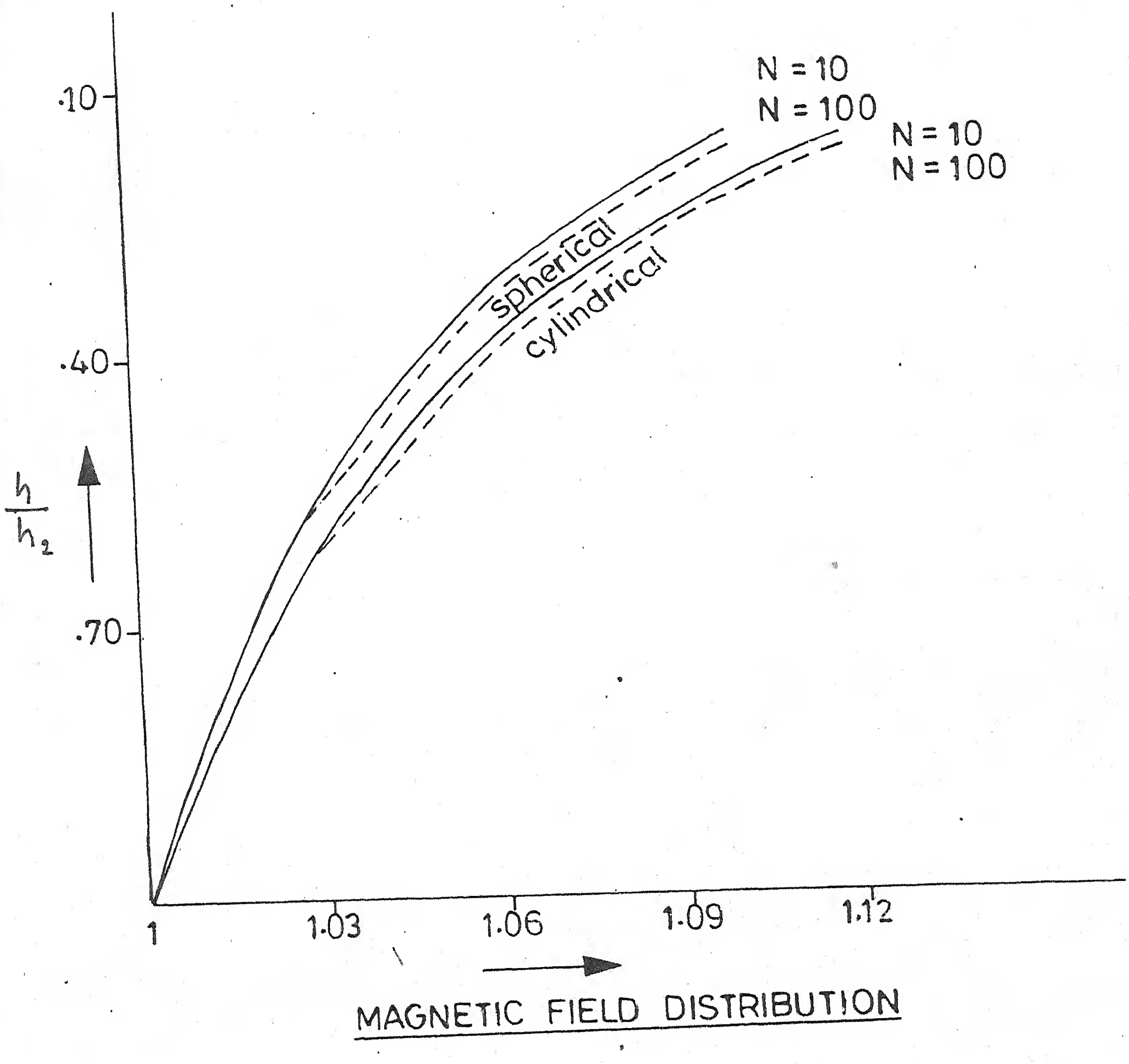
·10
·40
·70
$\frac{P}{P_2}$
N = 10
N = 100
N = 10
N = 100
spherical
cylindrical
1
1.03
1.06
1.09
1.12
DENSITY DISTRIBUTION



PRESSURE DISTRIBUTION



q/q½
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
N = 10
N = 100
N = 10
N = 100
spherical
cylindrical
1
1.03
1.06
1.09
1.12
HEAT FLUX DISTRIBUTION



N = 10
N = 100
N = 10
N = 100
spherical
cylindrical
·10
·40
·70
h/h2
1
1·03
1·06
1·09
1.12
MAGNETIC FIELD DISTRIBUTION

REFERENCES

(1) Lin, S.C :

J APPL. PHYS 25, 54 (1954)

(2) Ray,G. Deb :

Proc,Nal,Int.Sci.India; 23A 420 (1957)

(3) Sakurai,A :

J Phys Soc , Japan; 10,827 (1955)

(4) Rogers M.H :

Quart J.Mech. Appl Math, (1.41) (1958)

(5) Singh, J.B and Vishwakarma, P.R.:

Astrophys Space Science; 93,423 (1983)

(6) Zeldovich,YA.D And Raigers,YV.P :

Shock waves and Radiation Analytical in Annual Rew. of fluid mechanics 1,385 (1959)

(7) Wang,K.C.:

Phys. of Fluids 9,(1922)

CHAPTER III

SELF SIMILAR MEGNETO GAS DYNAMIC CYLINDRICAL SHOCK WAVES

(1) INTRODUCTION

A cylindrical wave of explosion with a shock surface as wave front produced on account of a sudden release of a finite amount of energy, expanding outwords in a conducting gas subjected to a magnetic field has been studied by Pai[1],Christer and Helliwele [2] and many others.Deb Ray [3] and Singh - Vishwakarma [4]have obtained similarity solution for strong cylindrical blast waves in a conducting non uniform medium .

The present chapter deals with the problem of the instataneous release of energy along a line in a gas cloud thunder in the effect of magnetic field where density varies with time. The motion of a gas is assumed to be adeabatic and total energy of the wave remains constant. A comparison has been made on the effects of the transverse and axial components of the magnetic field variables behind the shock surface. The transverse and axial components of the megnetic filed (h) are h_θ and h_z respectively .The numerical calculation have been done on DEC system 1090 computer by R.K.G.S programme.

(2) EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

Following Witham [5] , the equation of motion for one-dimentional unsteady flow of a perfect gas with transverse and axial megnetic field are

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \rho \frac{u}{r} = 0 \quad , \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left[\frac{\partial P}{\partial r} + \frac{1}{2} \frac{\partial}{\partial r} (H_\theta^2) + \frac{H_\theta^2}{r} \right] = 0 , \tag{2.2}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left[\frac{\partial P}{\partial r} + \frac{1}{2} \frac{\partial}{\partial r} (H_z^2) \right] = 0 , \tag{2.3}$$

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial r} - \frac{\tau P}{\rho} \left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right] = 0 , \tag{2.4}$$

$$\frac{\partial H_\theta}{\partial t} + H_\theta \frac{\partial u}{\partial r} + u \frac{\partial H_\theta}{\partial r} = 0 . \tag{2.5}$$

$$\frac{\partial H_z}{\partial t} + H_z \frac{\partial u}{\partial r} + H_z \frac{u}{r} + u \frac{\partial H_z}{\partial r} = 0 , \tag{2.6}$$

where P, u, ρ, H_θ, H_z are pressure, velocity, density, transverse magnetic field and axial magnetic field. Magnetic field permeability (μ) of the medium is taken to be unity through out the problem. All are function of r and t. τ is the ratio of specific heats.

Following Witham [5], we find the strong shock boundary conditions to be.

$$\rho_1 = \frac{\tau+1}{\tau-1}\,\rho_0\,,$$

$$P_1 = \frac{2}{\tau-1}\,\rho_0\,\dot{R}^2\,,$$

$$u_1 = \frac{2}{\tau-1}\,\dot{R}\,, \qquad (2.7)$$

$$H_{\theta 1} = \frac{\tau+1}{\tau-1}\,H_{\theta 0}\,,$$

$$H_{z1} = \frac{\tau+1}{\tau-1}\,H_{z0}\,,$$

where subscripts (1) and (0) denote the regions just inside and just outside the shock front respectively .

(3) SIMILARITY SOLUTIONS

In order to reduce the equations of motion to ordinary differential equation , we now introduce the following similarity transformation . Let the similarity variables η be in the form $\eta = r/R$, $R = R\,(t)$. (3.1).

The solution of the partial differential equation (2.1) to (2.6) ,in the form

$$u = \dot{R} \; V(\eta) \quad ,$$

$$\rho = \rho_0 \; g(\eta) \quad ,$$

$$P = \rho_0 \; \dot{R}^2 \; (\eta) \; ,$$

$$H_\theta = \rho_0^{1/2} \; \dot{R} \, h_\theta \; (\eta) \quad , \qquad (3.2)$$

$$H_z = \rho_0^{1/2} \; \dot{R} \, h_z \; (\eta) \quad ,$$

where v, g, t, h_θ , and h_z are the function of η. The scales R, ρ_0 are time independent in some manner yet unknown .

By substituting the relation (3.2) in to equation (2.1) - (2.6), taking account of the definition of similarity variables (3.1) and using of the relation of the type to transform the derivatives

$$\frac{\partial \rho}{\partial t} = g \, \dot{\rho}_0 - \rho_0 g' \eta \; \frac{\dot{R}}{R} \quad ,$$

$$\frac{\partial \rho}{\partial r} = \rho_0 \; \frac{g'}{R} \quad , \qquad (3.3)$$

$$\frac{\partial H_\theta}{\partial t} = \frac{h}{\rho_0^{1/2}} (2 \, \ddot{R} \, \rho_0 + \dot{R} \, \dot{\rho}_0) - \rho_0^{1/2} - \frac{\dot{R}^2}{R} \eta \, h' = \frac{\partial H_z}{\partial t} \quad ,$$

$$\frac{\partial H_\theta}{\partial r} = \rho_0^{1/2}\ \frac{\dot R}{R}\ h' = \frac{H_z}{r} \quad . \tag{3.3}$$

The dot represents differentiation of scales with respect to time and prime denote differentition of functions with respect to similarity variables. After using the relation, differential equation becomes

$$\frac{\dot\rho_0}{\rho_0} + \frac{\dot R}{R}\left[V' + (V - \eta)\frac{g'}{g} + \frac{V}{g} \right] = 0 \quad , \tag{3.4}$$

$$\frac{R}{\dot R}\left[\frac{\ddot R}{\dot R} + \frac{\dot\rho_0}{2\rho_0}\right] + (V - \eta)\frac{h'_\theta}{h_\theta} + V' = 0 \ . \tag{3.5}$$

$$\frac{R\ddot R}{\dot R^2} V + [V - \eta]\, V' + \frac{1}{g}\left\{ f' + h_\theta \left(h'_\theta + \frac{h_\theta}{\eta} \right) \right\} = 0 \ . \tag{3.6}$$

$$\frac{R\ddot R}{\dot R^2} V + [V - \eta]\, V' + \frac{1}{g}\left(f' + h_z\, h'_z \right) = 0 \ . \tag{3.7}$$

$$\frac{R}{\dot R}\ \frac{d}{dt}\left[\log \dot R^2 \rho_0^{1-\tau} \right] + (V - \eta)\left(\frac{f'}{f} - \frac{g'}{g} \right) = 0 \ . \tag{3.8}$$

Inorder that similarity solutions (3.2) be meaningful , it is necessary that variable t and η in equation (3.4 - 3.8) must be separable , this possible if

$$\frac{R\ddot{R}}{\dot{R}^2} = \text{constant} ,$$

and

$$\frac{\dot{\rho}_0}{\rho_0} = \text{constant} :$$

$$R = At^{\alpha} , \qquad (3.9)$$

$$P_0 = Bt^{\beta} , \qquad (3.10)$$

where A , B, α and β are constants . Thus all the scales in the self similar motion have a power law dependence on time . By use of relation (3.9) and (3.10) equations (3.4) - (3.8) take the non dimensional form of ordinary differential equation are as ,

$$\frac{\beta}{\alpha} + V' + (V - \eta) \frac{g'}{g} + \frac{V}{\eta} = 0 , \qquad (3.11)$$

$$\frac{2(\alpha - 1) + \beta}{2\alpha} + (V - \eta) \frac{h'_\theta}{h_\theta} + V' = 0 . \qquad (3.12)$$

$$\frac{2(\alpha-1)+\beta}{2\alpha} + (V-\eta)\frac{h_z'}{h_z} + V' + \frac{V}{\eta} = 0 \,, \qquad (3.13)$$

$$\frac{(\alpha-1)}{\alpha}V + (V-\eta)V' + \frac{1}{g}\{f' + h_\theta(h_\theta' + \frac{h_\theta}{g})\} = 0, \qquad (3.14)$$

$$\frac{(\alpha-1)}{\alpha}V + (V-\eta)V' + \frac{1}{g}\{f' + h_z h_z'\} = 0 \,, \qquad (3.15)$$

$$\frac{2(\alpha-1)+(1-\gamma)}{\alpha} + (V-\eta)\{\frac{f'}{f} - \gamma\frac{g'}{g}\} = 0 \,. \qquad (3.16)$$

The total energy of the flow as

$$E = 2\pi\int_0^R \frac{1}{2}\rho u^2 + \frac{P}{\gamma-1} + \frac{1}{2}(H_\theta^2 \text{ or } H_z^2)\, r\,dr \,. \qquad (3.17)$$

Equation (3.2) , (3.9) and (3.10) reduce the above equation in the form .

$$E = 2\pi B A^4 \alpha^2 t^{4\alpha+\beta-2}\int_{\eta_0}^{1} \frac{1}{2}gV^2 + \frac{f}{\gamma-1} + \frac{h_\theta^2}{h_\theta} \text{ or } (\frac{h_z^2}{h_z})\, 3\eta\, d\eta \,. \qquad (3.18)$$

Since the total energy of the gas in constant so we have

$$\beta = 2 - 4\alpha \,. \qquad (3.19)$$

It is clear from the equation (3.19) that uniform expansion of wave takes place when $\beta = -2$. The solution have their physical significance only when $-2 \le \beta \le 0$

After using similarity transformation (3.2) the jump condition (2.7) becomes

$$g(1) = \frac{\tau+1}{\tau-1},$$

$$f(1) = \frac{2}{\tau+1}, \qquad (3.20)$$

$$h_\theta(1) = M_A^{-1}\frac{\tau+1}{\tau-1},$$

where M_A is the Alfven Mach numbers $M_A \rho^{1/2} \frac{\dot{R}}{H_0}$

(4) RESULT AND DISCUSSION

The calculation of results have been obtained in the following non dimensional form

$$\frac{u}{u_1} = \frac{\tau+1}{2}\, u(\Pi), \qquad (4.1)$$

$$\frac{\rho}{\rho_1} = \frac{\tau-1}{\tau+1}\, g(\Pi), \qquad (4.2)$$

$$\frac{P}{P_1} = \frac{\gamma+1}{2} f(\eta) \quad , \qquad (4.3)$$

$$\frac{H_\theta}{H_{\theta 1}} = \frac{\gamma-1}{\gamma+1} M_A \, h_\theta(\eta) \quad , \qquad (4.4)$$

$$\frac{H_z}{H_{z1}} = \frac{\gamma-1}{\gamma+1} M_A \, h(\eta) \quad . \qquad (4.5)$$

Numerical result for certain choices of parameters are reproduced in the form of graphical form . The calculation of the result are for following values of parameters .

(i) $\gamma=1.4$, $\alpha=0.25$, $M_A=10$, $\beta=1$

(ii) $\gamma=1.4$, $\alpha=0.25$, $M_A=10$, $\beta=0$

In this problem, for the shock which is produced by a line explosion moves in conducting gas.we have discussed two cases and makes a comparision between them .

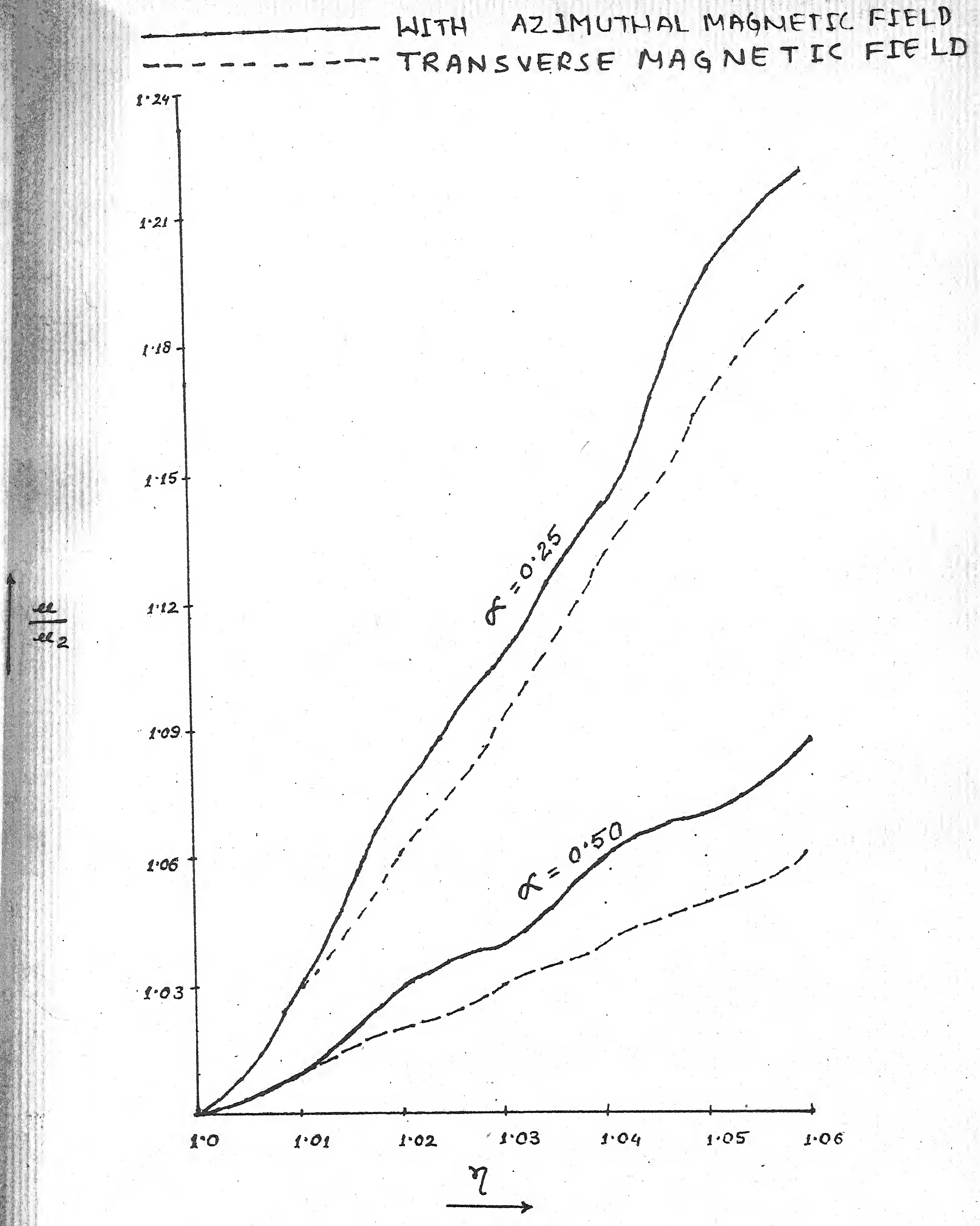
Case I

Propagation of cylindrical shock wave with azimuthal magnetic axial field .

Case II

Propagation of cylindrical shock wave with transverse magnetic field .

A comparision has also been made for both the above cases . It is observed that when the shock radius is contrating that is α = .25, ther is sharp increase in velocity , density, pressure and magnetic feild of the fluid particles behind the shock surface as compared to α = .50 , when the contraction into shock radius is less .



VELOCITY DISTRIBUTION

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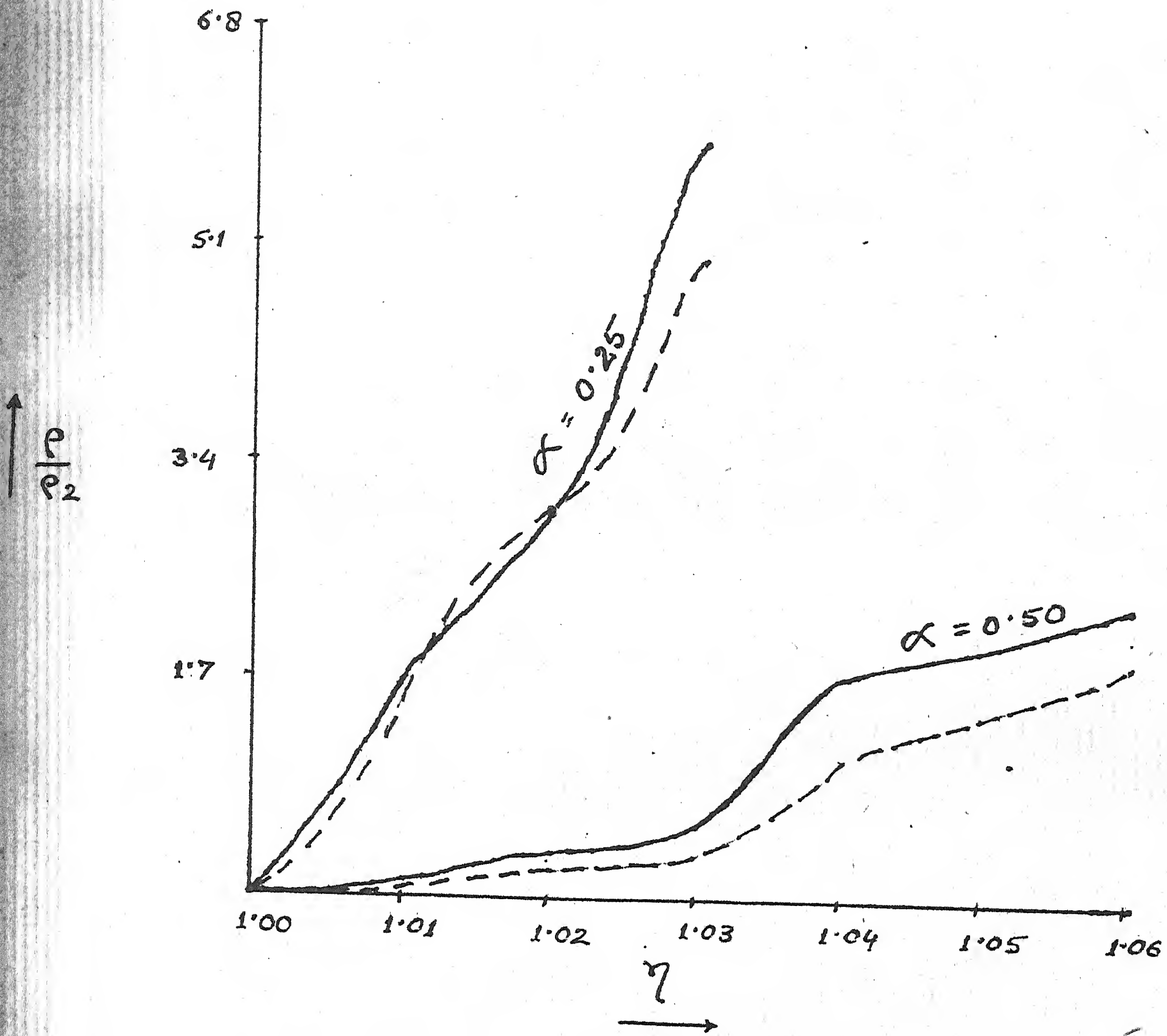
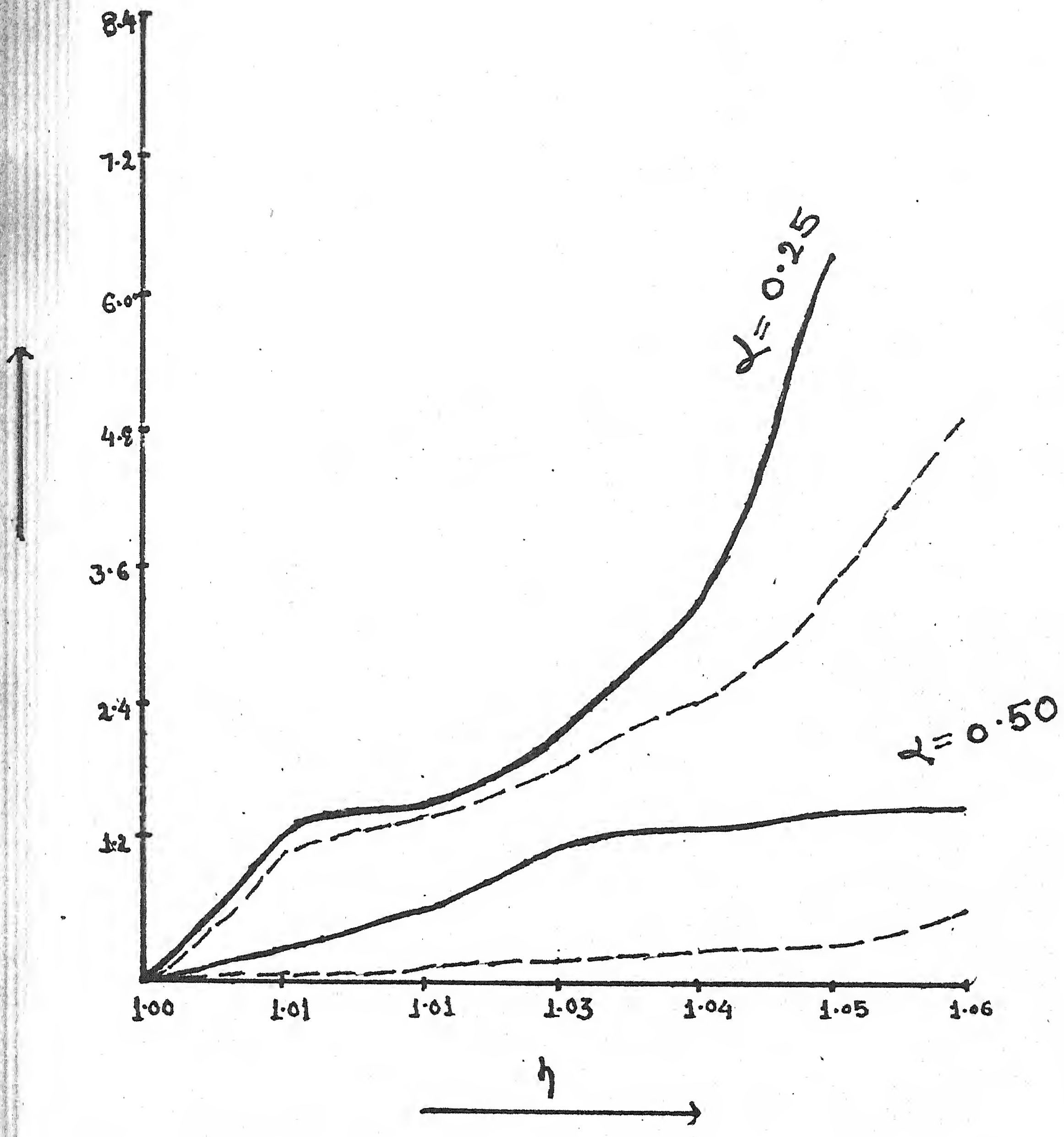
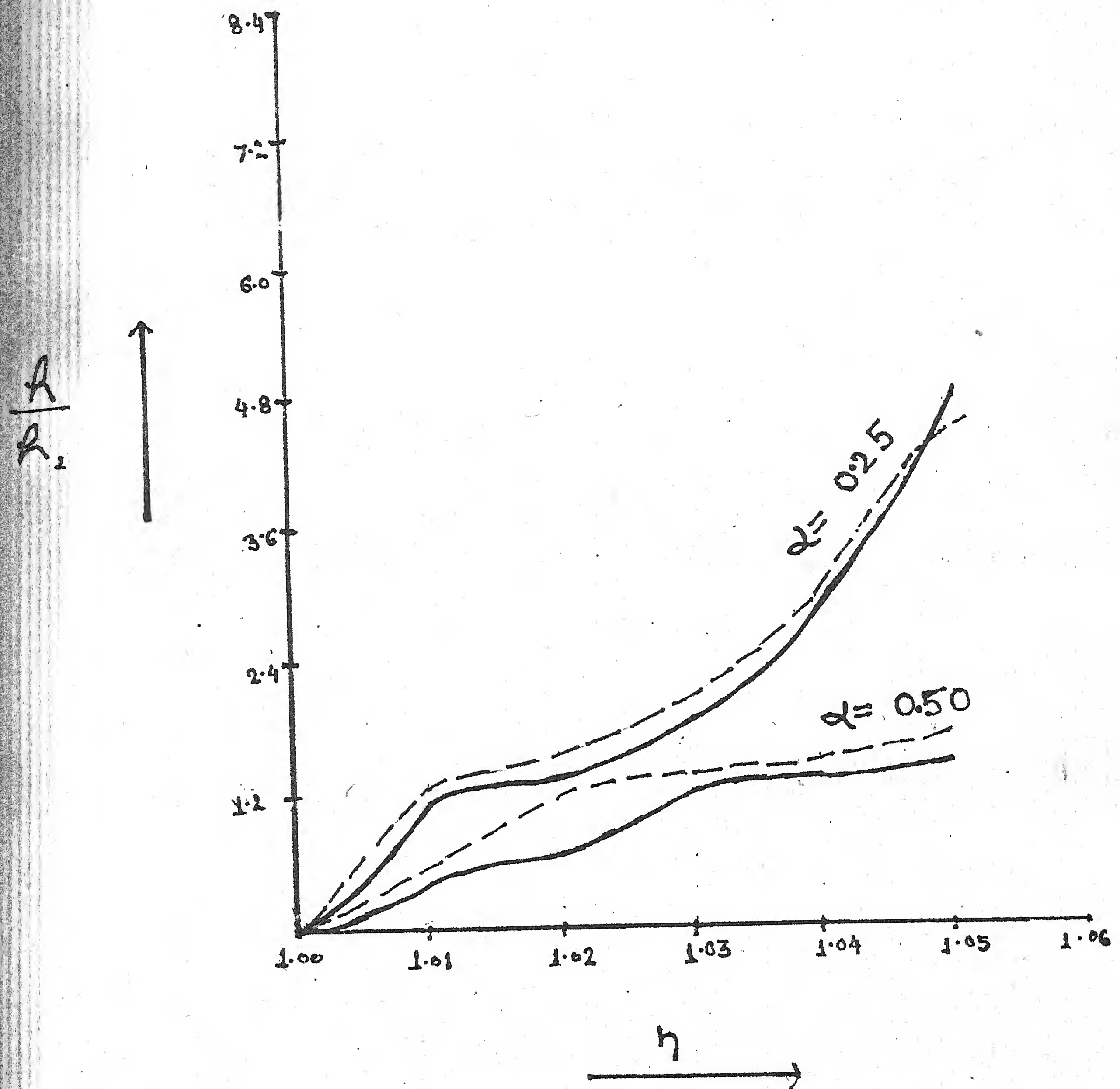


DENSITY DISTRIBUTION

1[B]



PRESSURE DISTRIBUTION



MAGNETIC FIELD DISTRIBUTION

REFERENCES

(1) S.I. Pai

Proc. fourth cong. , Appl Mech 89 , 134 (1958)

S.I.Pai

Magnetogasdynmics and plasma dynamics ,springer verlag (1962)

(2) Christer , A.H. and Helliwell . J.B:

J fluid Mech 39 , 705 (1969)

(3) Deb Ray , G :

Phys fluids 16 . 559 (1973)

(4) Singh J.B : and Vishwakarma P.R :

Astrophys and space sci 93 . 423 (1983)

(5) Witham , G.B :

J Fluid Mech 4 , 337 (1958)

CHAPTER IV

PROPAGATION OF SPHERICALLY SYMMETRICAL DISCONTINUITIES WITH INCREASING ENERGY IN GENERALIZED ROCHE MODEL

(1) INTRODUCTION

In the earlier investigation , the self - similar solution driven out by a sudden point explosion of core of the genera - lized Roche model are investigated by Carrus etal [1] . Rogers [2] has discussed methods for obtaining analytical solution of the same problem . Runga Rao and Purohit [3] has studied the self similar isothermal flow in generalized Roche model. Roseenau [4] has attempted the self similar adiabatic flow behind spherical shock wave in the prescence of magnetic field .One of the basic assumption of their work is that the total energy contain behind the shockfront is constant . Deb Ray [5] has reviwed the Roche model and obtained the exact non similarity solution taking total energy of the wave non constant .

In this chapter, the similarity solution in the Generalized Roche model has been developed,when the radiation heat flux is more important then the radiation pressure and radiation energy . The effect of magnetic field has also taken into account . The unsteady model of Roche consists of a gas distributed with spherical symmetry arround a nucleus having a large mass (m) . It is assumed that the gravitating effect of gas it self can be neglected compared with the attraction of heavy nucleus .

(2) EQUATION OF MOTION AND BOUNDARY CONDITIONS

The equation of continuity momentum . field and energy in the Generalized Roche model are .

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \quad . \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{1}{2}\frac{\partial}{\partial r}(h^{2}) + \frac{1}{\rho}\frac{h^{2}}{r} + \frac{Gm}{r^{2}} = 0 \quad . \qquad (2.2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} = 0 \quad . \qquad (2.3)$$

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial r} - \frac{\tau P}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{r}\right) + (\tau-1)\frac{1}{r}\frac{\partial}{\partial r}(r.F) = 0 \quad . \qquad (2.4)$$

where u ,P,ρ.h and F are the velocity , pressure , density , magnetic field and radiation heat flux , at radial distance (r) from the centre of core at time (t) ; G be the gravitational constant .The magnetic permeability of the medium has taken to be unity through out the problems . The equation of state for ideal gas is given by

$$P = \Gamma \rho\, T \quad , \qquad (2.5)$$

where Γ is the gas constant .

Also assuming local thermodynamic equilibrium and taking Rosseland's diffusion approximation [Zeldovich Raizors (6)]

$$F = -\frac{C\mu}{3}\frac{\partial}{\partial r}(\sigma T^4) , \qquad (2.6)$$

where $\sigma C/4$ is the Stefan - Boltzmann constant , C the velocity of light and μ the mean free path of radiation is a fanction of density and absolute temperature T .

Following Wang [7]

$$\mu = \mu_0 \rho^{\alpha} T^{\beta} . \qquad (2.7)$$

where μ_0 , α, β, being constant . In the self similar model the total energy of the wave is dependent of time as

$$E = B t^{q} , \quad (q \le 0) , \qquad (2.8)$$

where B and q are constant

The flow variables immediately ahead of shock denoted by suffix (1) are

$$U_1 = 0 , \rho_1 = AR^{-w} , \quad (0<W<2) , \qquad (2.9)$$

where A is a constant and R denotes the radius of the shock surface ahead the shock , the magnetic field distribution is

$$h = CR^{-\beta} , \quad 2\beta = W+1$$

and pressure distribution ahead the shock .

$$P_1 = \frac{AGMR^{-(1+w)}}{(W+1)} + \frac{(1-\beta)}{\beta} C^2 R^{-2\beta} , \tag{2.11}$$

where C, W and β are constant . The Rankine Hugoniot shock conditions headed by an isothermal shock is

$$\rho_2 (V - u_2) = \rho_1 V = m_s , \tag{2.12}$$

$$P_2 + \frac{h_2^2}{2} - P_1 - \frac{h_1^2}{2} = m_s u_2 , \tag{2.13}$$

$$E_2 + \frac{P_2}{\rho_2} + \frac{h_2^2}{\rho_2} + \frac{1}{2} (V - u_2)^2 - \frac{F}{m_s} = E_1 + \frac{P_1}{\rho_1} + \frac{1}{2} V^2 + \frac{h^2}{\rho_1} \tag{2.14}$$

$$h_2 (V - u_2) = h_1 V , \tag{2.15}$$

$$T_1 = T_2 , \tag{2.16}$$

where suffix 2 , denotes the flow variable just behind the shock and 1 denotes flow variable just ahead the shock .m_s denotes the mass perunit area across the shock and V be the shock velocity and

given by

$$V = \frac{dR}{dt} \quad . \qquad (2.17)$$

3. TRANSFORMATION OF EQUATIONS OF MOTION

In order to reduce the equation of flow to ordinary differential equation we now introduce the following transformations . Following Sedov [7]

$$\eta = (\alpha\, MG\,)^{-1/3}\; r t^{-\sigma} \quad , \qquad (3.1)$$

$$\text{where} \quad \sigma = \frac{2}{3} \quad = \frac{2+q}{5-W} \quad ,$$

$$q = \frac{2}{3} \;(2 - W) \quad . \qquad (3.2)$$

and the limit of q and w are

$$0 \le q \le 4/3 \text{ and } 0 \le W \le 2 \quad . \qquad (3.3)$$

We see the solution of equation (2.1) - (2.4) in the form

$$u = \frac{r}{t} \; V(\eta) \quad , \quad \rho = \frac{AMG\; t^{2}}{r^{W+3}} \; R(\eta) \quad ,$$

$$P = \frac{AMG}{r^{W+1}} P(\eta) \quad , \quad h = \frac{(AMG)^{1/2}}{r^{W+1}} h(\eta) \quad , \qquad (3.4)$$

$$F = \frac{AMG}{r_t^{W+1}} F(\eta) \quad ,$$

using equation (3.1) in the equation (2.6) with the help of equation (2.5) we obtian

$$\alpha = \frac{W}{W+1} \quad \text{and} \quad \beta = \frac{-(5W+7)}{2(W+1)} \quad .$$

The equation (2.1) – (2.4) and the equation (2.5) are then transformed with the help of the relations (3.1) and (3.3) to following form

$$V(\eta) = \frac{\frac{1}{N}\frac{F(\eta)R^{\beta-\alpha+4}}{P(\eta)^{\beta+3}}(V-\sigma)-(V-\sigma)\left[2\eta p+(W+1)(p+H^2)-H^2R-\frac{R}{\alpha\eta^3}\right]+(W+1)Vp-2p-\frac{(1-W)}{2}VH^2}{\left[\eta P-\eta R(V-\sigma)^2+\eta^2H^2\right]}$$

$$\text{where} \quad N = \frac{4}{3}\,\frac{C\mu_0\sigma}{\Gamma^{4+\beta}}\,(AMG)^{\alpha-1} \quad , \qquad (3.6)$$

is a dimension less parameter

$$\frac{R'(\eta)}{R(\eta)} = \frac{1}{(V-\sigma)}\left[\frac{(W+1)}{\eta}V(\eta) - V'(\eta) - \frac{2}{\eta}\right] , \qquad (3.7)$$

$$P'(\eta) = \frac{(w+1)}{\eta}\left[P + H^2\right] - \frac{H^2 R}{\eta} - \frac{R}{\alpha\,\eta^4} - \frac{VR(V-1)}{\eta} - H H' , \qquad (3.8)$$

$$F'(\eta) = \frac{1}{(\tau-1)}\left[\frac{\eta R P'}{R} - P'\right](v-\sigma) + \frac{2\tau P}{(\tau-1)\eta} \quad \frac{P\,v}{\eta(\tau-1)}\left[w(\tau-1)+3\tau-1\right]$$

$$- \frac{(1-w)}{\eta} F(\eta) . \qquad (3.9)$$

4. RESULT AND DISCUSSIONS .

The transformed jump condition at the shock front is given by

$$v(1) = \frac{2}{(4+w)}\left[1 - \frac{1}{\tau m^2} - \frac{1}{\tau m_A^2}\right] , \qquad (4.1)$$

$$R(1) = \frac{\tau M^2 M_A^2}{(M^2 + M_A^2)} , \qquad (4.2)$$

$$P(1) = \frac{4}{(4+W)} \quad \frac{M_A^2}{(M^2 + M_A^2)} \quad , \tag{4.3}$$

$$F(1) = \frac{-4}{(4+w)} \left[\frac{M_A^2 + M^2}{\tau M_A + M^4} - 1 \right] \quad , \tag{4.4}$$

$$H(1) = \frac{4}{(4+W)^2} \left[\frac{3}{\tau M^2} + \frac{2\tau M_A^4}{\tau M^2 \, (M_A^2 + M^2)} \right] \quad . \tag{4.5}$$

For exhibiting the numerical solution it is convenient to write the field variables in non dimensional form

$$\frac{u}{u_2} = \eta \, \frac{V(\eta)}{V(1)} \quad , \tag{4.6}$$

$$\frac{\rho}{\rho_2} = \frac{1}{\eta^{w+3}} \quad \frac{R(\eta)}{R(1)} \, , \tag{4.7}$$

$$\frac{p}{p_2} = \frac{1}{\eta^{W+1}} \quad \frac{P(\eta)}{P(1)}, \tag{4.8}$$

$$\frac{h}{h_2} = \frac{1}{\eta^{(w+1)/2}} \quad \frac{H(\eta)}{H(1)}, \tag{4.9}$$

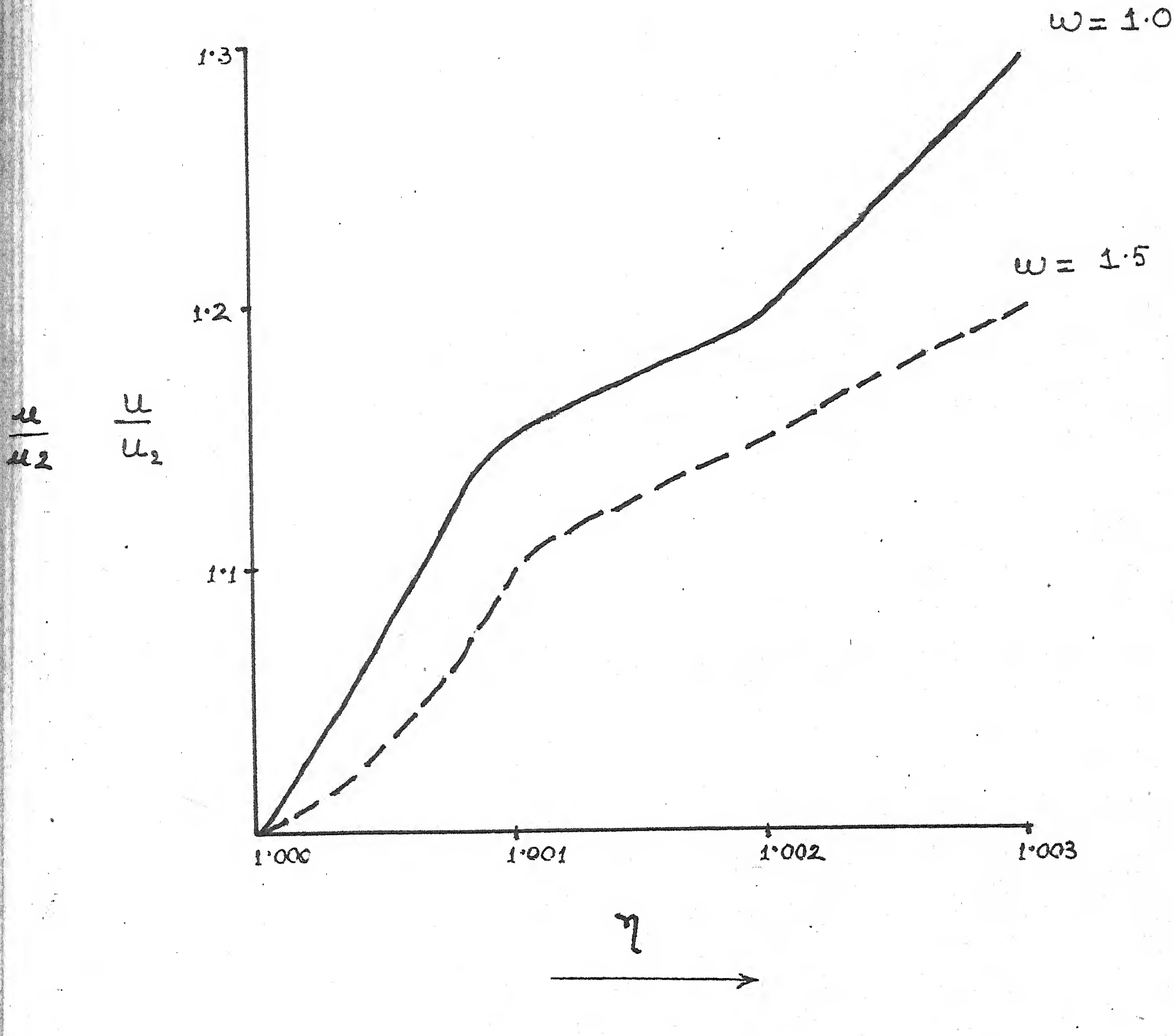
$$\frac{f}{f_2} = \frac{1}{\eta^{w}} \quad \frac{F(\eta)}{F(1)} . \tag{5.0}$$

The numerical integration is carried out on DEC-system 1090 computer installed at I.I.T. Kanpur by well known R.K.G.S programme for the values of w=1,1.5 .

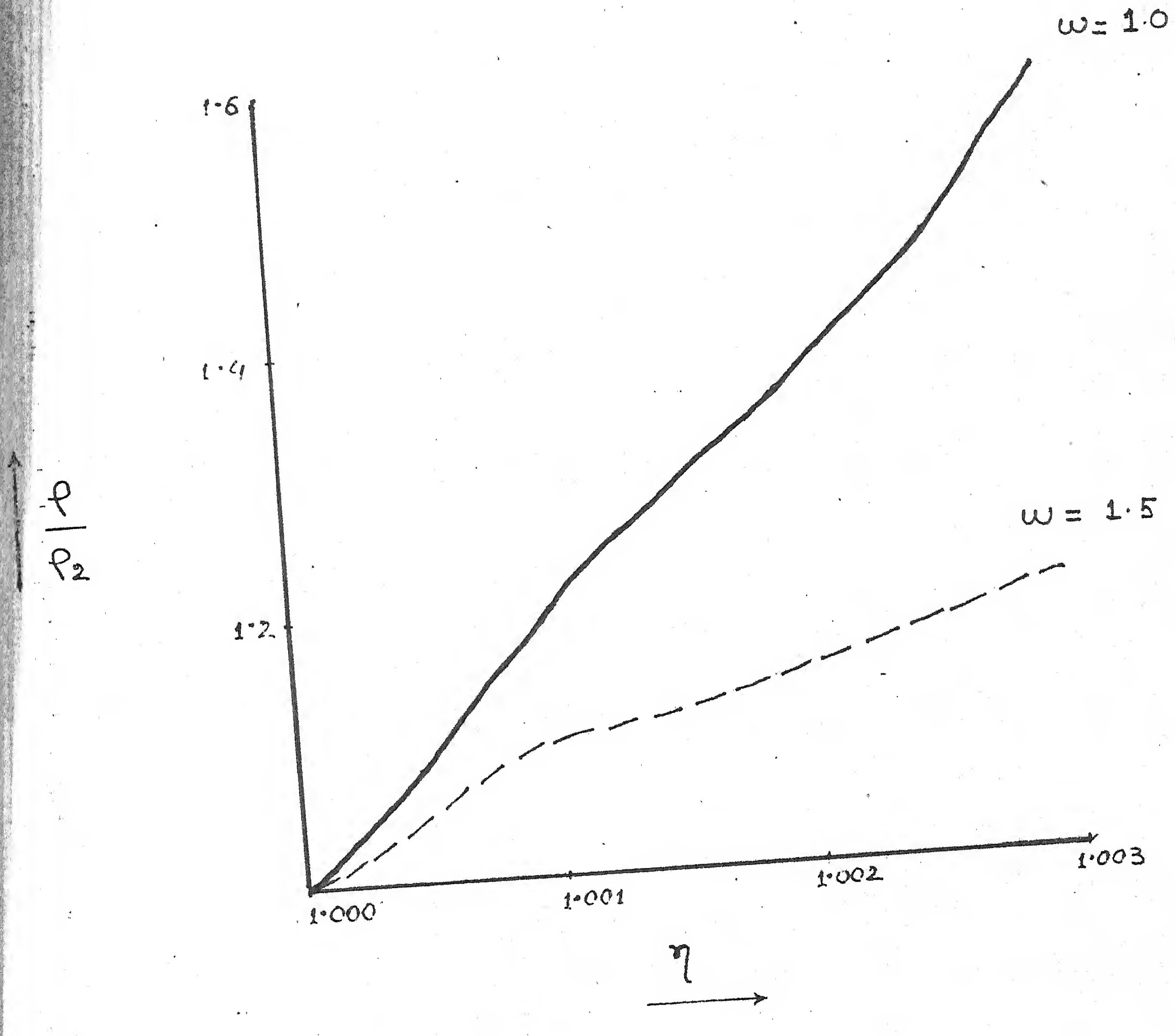
The other parameters are

$$M^2 = 15,\ M_A^2 = 25,\ \alpha = 1/3,\ \beta = \frac{-(7+5w)}{2(1+w)},\ N=10 .$$

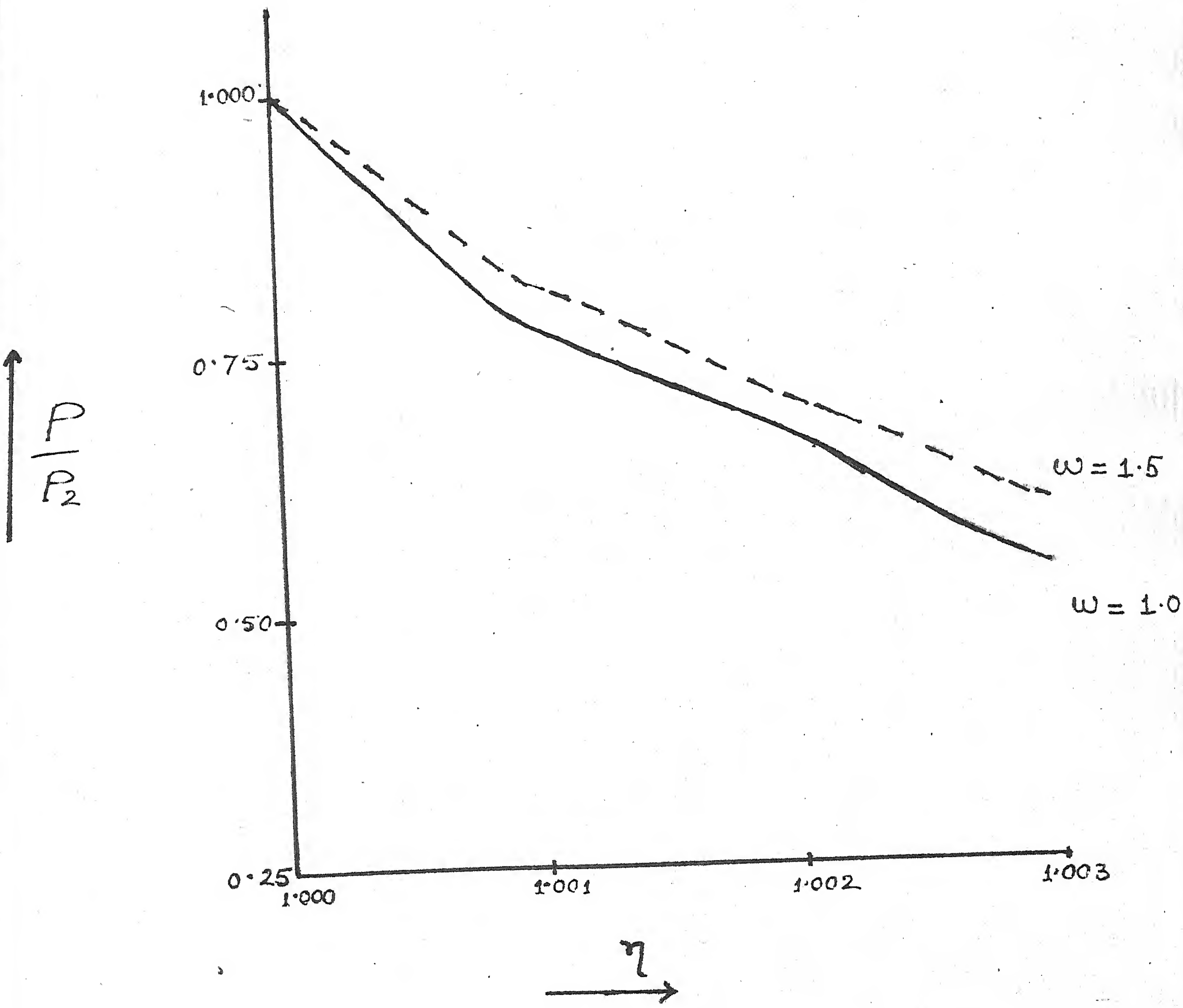
The nature of flow and field variables for adiabatic case are illustrated through graphs,it is clear from the graphs that velocity , density and pressure are minimum at shock front but increases rapidely towords the center of explosion in adiabatic case . Where as distribution of magnetic field and radiaton heat flux is maximum at shock front but decreases towords the centre of explossion.



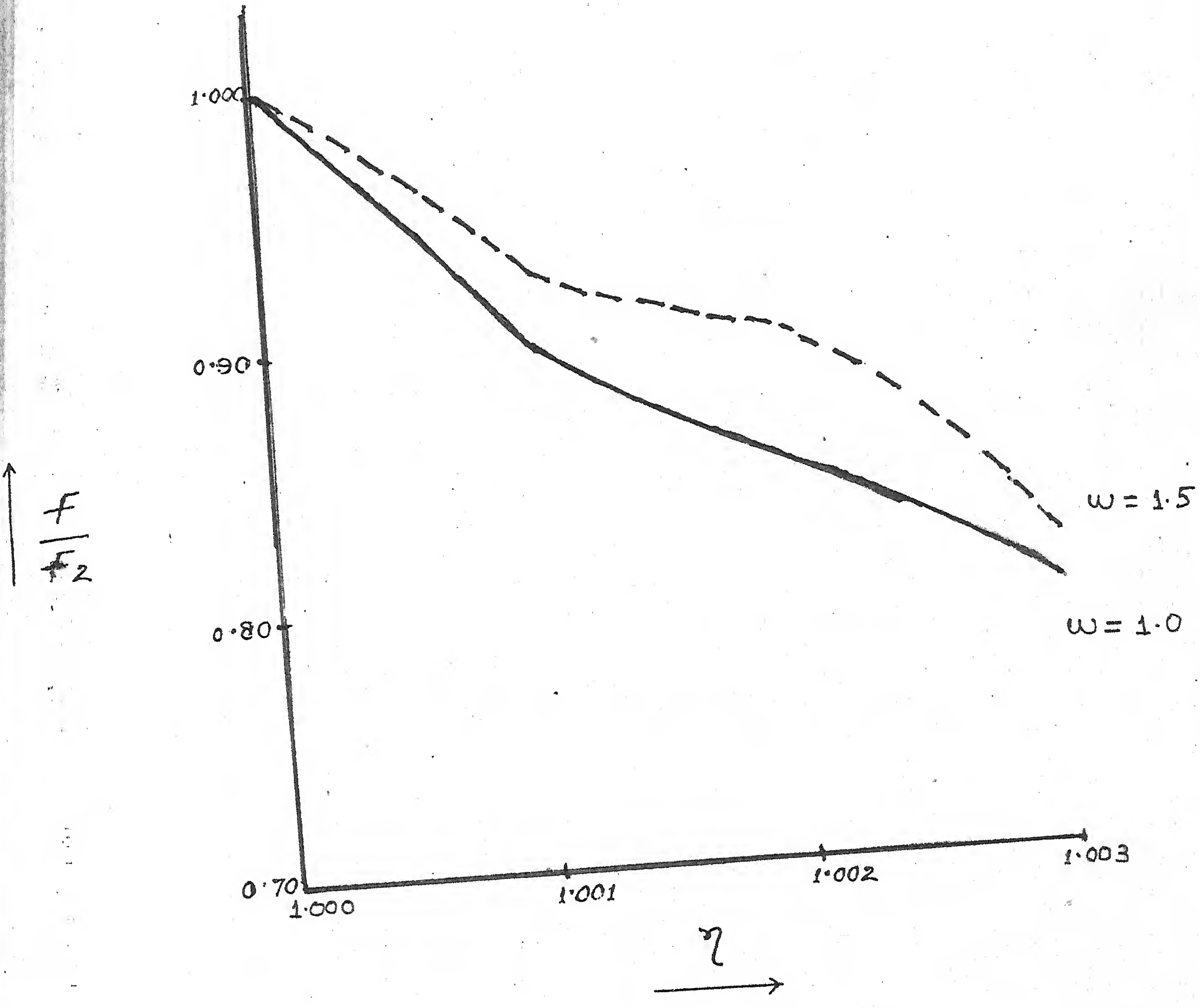
VELOCITY DISTRIBUTION



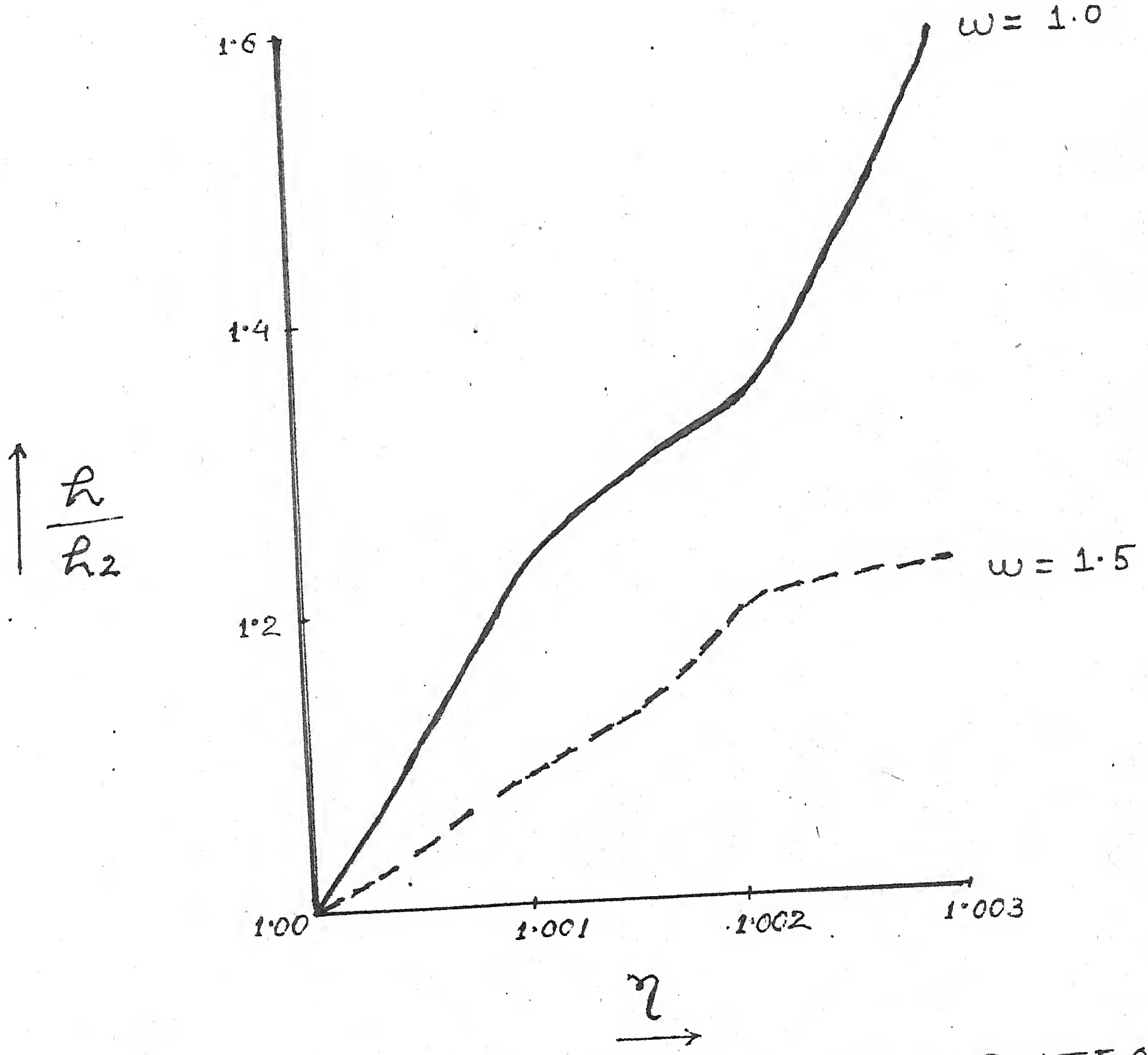
DENSITY DISTRIBUTION



PRESSURE DISTRIBUTION



RADIATION DISTRIBUTION



MAGNETIC FIELD DISTRIBUTION

REFERENCES

(1) Carrus etal P.A, Fox P.A, Hans. F and Kopal Z . ;
Astrophys . J 113,193 (1991)

(2) Rogers M.H. ;
Astrophys J . 145, 470 (1957)

(3) Runga Rao M.P and Purohit S.C . ;
Astron Astrophys ,23, 155 (1973)

(4) Rosenau P. ;
Phys Fluid,20,1097 (1977)

(5) Deb Ray G. ;
Bull Cal Math Soc.,6o9 ,225 (1977)

(6) YA.B. zel'dovich and YV.P. Raizer ;
Shock waves and Radiation, An article in Annul Review of fluid mechanics 11, 385 (1959)

(7) Sedov L I . ;
Similarity and Dimensional methods in machanics Academic Press Londan , (1959)

CHAPTER - V

ANALYSIS OF SELF SIMILAR MOTION IN THE THEORY OF STELLAR EXPLOSION

(1) INTRODUCTION

Carrus etal [1] and Sedov [2] were first to discuss the model of stellar exolosion in which a star is considered to be a perfect self gravitating gas. The distribution of gas at any moment of time spherically symmetric. In the present chapter a anyalytical solution of the classical model of stellar explosion has been invastigated. A number of new solutions has been obtained in which radial oscillation of gas occur after the shock wave passes. Taking Newtonian gravitation into account a thorough analytical study of self similar motion of a gas dynamics under the effect of magnetic field is developed in the theory of stellar explosion which was earlier applied by Novikov[3] and Bogoyavlenky[4]. It is supposed that originally the star is in equilibrium state the gas density ρ, the pressure P, the mass M of the gas within sphere of radious r, the radial gas velocity u, the magnetic field h have the form

$$\rho_1 = Ar^{-w}, u_1 = 0, M_1 = \frac{4\pi Ar^{3-w}}{3-w},$$

$$h_1 = Cr^{-k}, 2k = w+1, \tag{1.1}$$

$$P_1 = \frac{2\pi A^2 G}{(3-w)\ (w-1)} r^{2(1-w)} + \frac{C^2\ (1-k)}{2K} r^{-2K} .$$

where A.w and G are constant. As a result of enerov libration at the centre of svmmetrv r=O: a shock wave travels out from the centre. The motion of oas behind the shock front self similar and adiabatic

$$\frac{d}{dt}\left(\frac{P}{\rho^{\tau}}\right)=0 \quad ,\tau>1 \quad . \tag{1.2}$$

(2) EQUATION OF MOTION AND BOUNDARY CONDITION

The fundamental differential eouation sionifvino the conservation laws of sphericallv svmmetric motion in a self oravitatino oas.where the maonetic field is sionificant are Summer[5]

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{\rho}{r^2} \frac{\partial}{\partial r} (ur^2) = 0. \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{1}{\rho} \frac{h^2}{r} + \frac{GM}{r^2} = 0, \tag{2.2}$$

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial r} - \frac{\tau P}{\rho} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0 \quad , \tag{2.3}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + h\frac{u}{r} = 0 \; . \qquad (2.4)$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \; . \qquad (2.5)$$

where r,t,u,ρ,P,M are radial distance from centre , time , velocitv ,densitv , oressure and mass contained in a sohere of radius r

The disturbance is headed bv an isothermal shock with condition

$$\rho_2 (v-u_2) = \rho_1 v = m_s \; . \qquad (2.6)$$

$$p_2 + \frac{h_2^2}{2} - p_1 - \frac{h_1^2}{2} = m_s u_2 \; . \qquad (2.7)$$

$$h_2 (v-u_2) = h_1 v \; . \qquad (2.8)$$

$$E_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(v-u_2)^2 + \frac{h_2^2}{\rho_2} = E_1 + \frac{p_1}{\rho_1} + \frac{1}{2}V^2 + \frac{h_1^2}{\rho_1} \; , \qquad (2.9)$$

$$T_1 = T_2 \; , \qquad (2.10)$$

$$M_1 = M_2 \; , \qquad (2.11)$$

where suffix 1 and 2 denotes the flow variables just ahead and just behind the shock front respectively denotes the mass flux perunit area across shock and V denote velocity of shock , is ginen by

$$V = \frac{dr}{dt} \quad . \tag{2.12}$$

According to Sedov [5] , the total energy of gas between spheres of radii r_1 , and r_2 in equilibrium is

$$E = 8\pi^2 GA^2 \frac{1 - 2\,(w-1)\,(\tau-1)\,r^{5-2w}}{(\tau-1)\,(w-1)\,(3-w)\,(5-2w)} , \tag{2.13}$$

$$E = \frac{32\pi\,(4-3\tau)}{3(\tau-1)} \log \frac{r_2}{r_1} \quad .$$

If $w < 5/2$, the total energy E enclosed within the sphere of radii r is finite if $w \geq 5/2$ it is infinite but $E < 0$ if $\tau > \tau_0$ and $E \geq 0$ if $\tau \leq \tau_0$ where $\tau_0 = 2w - 1 / 2(w-1)$. The solutions of the problem of stellar explosions for $w \geq 5/2$ in the class of gas motion considered can only be applied to certain astrophysical solutions because of divergence of E at the lower limit . These solutions are regarded as intermediat asymptotic solutions valid out side a small neighbourhood of $r = 0$, solutions with $\tau \leq \tau_0$ $(E \geq 0)$ discribe the break down of unstable stellar equilibrium . The law of energy liberation in a Self similar solutions has the form (Bogoyaslenky [4])

$$E = \alpha G^{(5-w)/w} A^{5/w} t^{2[5-2w]/w} . \qquad (2.14)$$

Evidentlv E is indeoendent of the time for $w = 5/2$ or $\alpha=0$. The constant α is calculated from the solution it self and in some case turn out to be infinite . The corresoondino solutions then provided asvmototic form for a verv intense exolosion . The followino new results mav directlv be deduced bv

(1) For $\tau < 4/3$. $w = 5/2$ and also for $\tau < 2(w-1)/3$ there exist no solution with a vacuum formino withen the oas

For $\tau < 4/3$. $w = 5/2$. $M=1$. damoed Oscillation of the oas accur after the shock wave oasses which are connected with the limit of dvnamic svstem

(2) For $\tau < 4/3$. $w = 5/2$ all solutions have a soherically vacuum of increasino radii which forms about the centre . the oas monotonicallv soreadino out from it .

(3) For $\tau < 1/4$. $w = 5/2$. $M=1$. the oas returns to eouilibrium after the shock wave oasses but in case when

$$\tau < \tau_1 = \frac{4[3 + (2w-5)^2]^{-1}}{8(w-1)}$$

reoeated damoed oscillation taken olace .

(3) SIMILARITY SOLUTIONS

The similaritv variables which reduce the equation governino the flow to ordinary differential equation is taken as

$$\rho = \frac{1}{G t^{2}} R(\eta) \; .$$

$$P = \frac{r^{2}}{(G t)^{4}} P(\eta) \; .$$

$$M = \frac{r^{3}}{G t^{2}} M(\eta) \; . \qquad (3.1)$$

$$V = \frac{r}{t} V(\eta) \; .$$

where $\eta = r \, (AGt)^{-1/W}$.

By using relation (3.1) the differential equation are transformed as

$$- V + V^{2} + \eta \frac{\partial V}{\partial \eta} \left[V - \frac{2}{W} \right] + \frac{2p}{r} + \frac{\eta}{R} \frac{\partial P}{\partial \eta} + \frac{2N^{2}}{R}$$

$$+ \frac{N}{R} \frac{\partial N}{\partial \eta} + M = 0 \; . \qquad (3.2)$$

$$-\left[\frac{2\eta}{W} \frac{\partial p}{\partial \eta} + 4p \right] V \, (\eta) \left[2p + \eta \frac{\partial p}{\partial \eta} \right] - \frac{\tau p}{R} \left[-2 \left(\frac{\eta}{W} \frac{\partial R}{\partial \eta} + R \right) + \eta \, V \frac{\partial R}{\partial \eta} \right.$$

$$= 0 \; . \qquad (3.3)$$

$$[-2N - \frac{2\eta}{W}\frac{\partial N}{\partial \eta}] + V\eta\frac{\partial N}{\partial \eta} + 3VN + \cap N\frac{V}{\eta} = 0 \quad , \qquad (3.4)$$

$$-2[\frac{\eta}{W}\frac{\partial V}{\partial \eta} + R\,] + \eta V\frac{\partial R}{\partial \eta} + [3VR + R\frac{\partial V}{\partial \eta}] = 0 \quad . \qquad (3.5)$$

and appropriate transformed jump conditions are

$$V(1) = \frac{2}{3}\,\Omega \quad ,$$

$$R(1) = \frac{V}{1-\Omega} \quad ,$$

$$P(1) = \frac{2V}{9}\,[\Omega + \frac{1}{\tau M^2} - \frac{\Omega\,(2-\Omega)}{2(1-\Omega)^2}\,M_A^{-2}\,] \; ,$$

$$N(1) = \frac{2}{3}\,(V)^{\frac{1}{2}}\,[\frac{1}{1-\Omega}]\,M_A^{-1} \quad , \qquad (3.6)$$

where

$$\Omega = (\,1 - \frac{2}{\tau M^2} - \frac{M_A^{-2}}{4}) - [\,(\frac{1}{2\tau M^2} + \frac{M_A^{-2}}{4})^2 + \frac{M_A^{-2}}{2}\,] \; ,$$

and

$$V = \frac{9}{2}\,\tau M^2\,[2(1+W) - \frac{\tau M^2}{M_A^2}(1-W)\,]^{-1} \; ,$$

To Investigate numerical flow solution we write flow variable in a non dimention form

$$\frac{u}{u_2} = \Pi \frac{V(\Pi)}{V(1)} ,$$

$$\frac{\rho}{\rho_2} = \frac{R(\Pi)}{R(1)} , \qquad (3.7)$$

$$\frac{P}{P_2} = \Pi^2 \frac{P(\Pi)}{P(1)}$$

$$\frac{h}{h_2} = \Pi \frac{N(\Pi)}{N(1)} ,$$

$$\frac{M}{M_2} = \Pi^3 \frac{M(\Pi)}{M(1)} .$$

DISCUSSION

Above relations show the distribution of velocity , density, pressure , magnetic field and mass distribution in the stellar model , when magnetic field is applied . A numerical approximation may also be obtained which may illustrate the behaviour of flow and field variables exactly beheind the surface of discontinuity.

CHAPTER - VI

AN EXACT SOLUTION OF NORMAL SHOCK WAVE OF VARIABLE STRENGTH ADVANCING INTO A REGION OF VARIABLE DENSITY

INTRODUCTION

The propagation of normal shock wave in a half space, due to point explosion or a line explosion has been discussed by many workers to some lavels of approximation .This approximation seems to have been originated by S.W. Yuan and various Co - workers ([1],[4]) ,who have further approximated the solution of the resulting equations by seeking a variety of quasi - similar solutions. Kynch [5] , Taylor [6] assumed the undisturbed density to vary according to some inverse power of the distance from the center of explosion and neglected the counter pressure . This chapter aims to find an analytical solution , representing a progressive wave applied to the problem of shock advancing into region where the pressure is constant but the density varies according as power law .

(2) FUNDAMENTAL EQUATIONS

In Eulerian co-ordinates , the equations of motions are given as [7]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{n u \rho}{x} = 0 , \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0 , \qquad (2.2)$$

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = 0 , \tag{2.3}$$

where n=0,1,2 for one dimensional plane, cylindrical and spherically symmetric flow : x the radial distance from the line of explosion : ρ density ; P pressure : u radial velocity and s is the entropy

The shock pressure is an electrically and thermally , we also have

$$P = C\rho^{\tau} e^{(\tau k s)} , \tag{2.4}$$

where $k = \frac{1}{C_p}$ and a sound speed is $a^2 = \frac{\tau P}{\rho}$.

ie., $a^2 = \tau C \rho^{\tau - 1} e^{\tau k s}$ with the entropy s and time t as independent variables, the equations (2.1)to(2.3) can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial u/\partial s}{\partial x/\partial s} + \rho \frac{nu}{x} = 0 , \tag{2.5}$$

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p/\partial s}{\partial x/\partial s} = 0 , \tag{2.6}$$

$$\frac{\partial x}{\partial t} - u = 0 \quad . \tag{2.7}$$

and also $$\frac{\partial P}{\partial s} = a^{2}(\rho s + K\rho) \quad , \tag{2.8}$$

Now, we try to find an exact and special solution of the equations of motion,

(2) SOLUTION TO THE PROBLEM

We choose a variable

$$\eta = t\, e^{(-ks/\alpha)} \quad , \tag{3.1}$$

where α is a constant to be determined. To obtain the solution, we can take the variables in the following form

$$x = t^{b} X(\eta), u = t^{b-1} U(\eta), \rho = t^{c} \Omega(\eta) \quad , \tag{3.2}$$

$$P = t^{2b+c-2} P(\eta) \quad .$$

Differentiating (3.2) with respect to η, equations (2.5), (2.8) can be reduced to ordinary differential equations for X, U, Ω, and p as

$$X'(C \Omega X' + \eta \Omega' X + n \Omega U) + U' \Omega X = 0 \quad , \tag{3.3}$$

$$P' + \Omega X' \{(b-1) U + \eta U'\} = 0 \quad , \tag{3.4}$$

$$U = b\,x + \Pi\, X' , \qquad (3.5)$$

$$\frac{\Pi P'}{P} = \tau \left(\Pi \frac{\Omega'}{\Omega} - \alpha \right) , \qquad (3.6)$$

where prime denotes differentiation .

We define the quantities Y and Z , such that

$$Y = \left(1 - \frac{bx}{U} \right)^{-1} , \qquad (3.7)$$

$$Z = \frac{A^2\, Y^2}{U^2} , \qquad (3.8)$$

where $A^2 = \frac{\tau P}{\Omega}$

using above new variables equation (3.1) & (3.4) can be written as

$$\Pi\, Z \left(\frac{P'}{P}\right) + \tau\, Y \left\{ b - 1 + \Pi \frac{U'}{U} \right\} = 0 . \qquad (3.9)$$

And as a consequence of (3.5) , equation (3.7) , becomes

$$\Pi \frac{X'}{U} = \frac{1}{Y} , \qquad (3.10)$$

Then by using variable (3.7)& (3.8), equation (3.8) to (3.6) and (3.9)& (3.10) reduce to following form

$$n \frac{X'}{X} = \frac{b}{Y-1} , \tag{3.11}$$

$$n \frac{U'}{U} = \frac{1}{Z-1} \left\{ Y \frac{(b-1)\, Z\, (C+\alpha)}{Y} - \frac{nbz}{Y-1} \right\} , \tag{3.12}$$

$$n \frac{\Omega'}{\Omega} = \frac{1}{Z-1} \left\{ \alpha Z - (b-1)\, Y + C + \frac{nbY}{Y-1} \right\} \tag{3.13}$$

$$n \frac{P'}{P} = \frac{T}{Z-1} \left\{ C+\alpha - (b-1)\, Y + \frac{nbY}{Y-1} \right\} , \tag{3.14}$$

$$n \frac{Y'}{Y} = b - (Y-1)\, n \frac{U'}{U} , \tag{3.15}$$

$$n \frac{Z'}{Z} = n \frac{P'}{P} + 2n \frac{Y'}{Y} - n \frac{\Omega'}{\Omega} - 2n \frac{U'}{U} . \tag{3.16}$$

The relation (3.12) to (3.16) finally give required exact solution of the differential equation as

$$\frac{dY}{dZ} = \frac{1}{Z} \; \frac{Y + (c+\alpha)Z + Y^2 (b-1) - Y\,Z\,(b+c+\alpha+nb)}{H}, \qquad (3.17)$$

where $H = Y(b-1)(\tau-1)-Z(2C+2b+\alpha)+2b+C-\tau(C+\alpha)+\dfrac{nbY}{Y-1}+(1-\tau-2\,Z)$

which atlhough not very simple but may determine the solution. In order to find the quantities $X, U, \Omega, \&\ P$, we have to solve (3.17) by quadrature for Y & Z from (3.17) , we derive

$$\frac{1}{\Omega}\frac{d\Omega}{dz} = \frac{(z-1)\,Z}{(\tau-1)+\tau\alpha-2b} - [\,(\tau+1)\,(b-1)\,Y\,] + (2b+2C+\alpha)\,Z$$

$$+\frac{nby}{Y-1}\,\{(\tau-1)+2\,Z\,\}. \qquad (3.18)$$

The shock problem suggests that c & α is such that $c+\alpha=0$. This leads to a simplification of (3.17). The value b=1 corresponds to the case of homentropic flow.

4.STRONG SHOCK CONDITIONS

If the shock is moving with a region of gas which is at rest under uniform pressure p, but a un-uniform temperature & density. Suppose that density distribution obeys the power low

$\rho_1(x)=\sigma x^m$ where σ & m are constants and the suffix 1 denotes the shock condition ahead the shock. The position of shock at the

time 't will be assumed to be given by $x = x_2 = \mu\, t^b$ where μ being shock constant. The shock speed is $v=dx/dt=b\mu t^{b-1}$ where suffx 2 denotes the quantities just behind the shock

so if, $\lambda^2 = \dfrac{\tau-1}{\tau+1}$, then

$$\frac{\rho_2}{\rho_1} = \frac{p_2+\lambda^2 p_1}{p_1+\lambda^2 p_2} \quad , \qquad (4.1)$$

$$p_2+\lambda^2 p_1 = (1-\lambda^2)\, p_1 v^2 \quad , \qquad (4.2)$$

$$u_2 = (1-\lambda^2)\left(v-\frac{a^2}{v}\right) \quad . \qquad (4.3)$$

Let $\eta=\eta_2$ represent the shock position, then eq^n (3.2) gives

$$P_2 = t^{2b+c+2}\, P(\eta_2) \; , \qquad (4.4)$$

$$\rho_2 = t^{c}\, \Omega(\eta_2) \; , \qquad (4.5)$$

$$u_2 = t^{b-1}\, U(\eta_2) \; , \qquad (4.6)$$

we may also write

$$\rho_1 = \sigma \mu^m t^{bm} \quad , \tag{4.7}$$

$$\text{and} \quad a_1^2 = \frac{\tau p_1}{\rho_1} = \left(\frac{\tau p_1}{\sigma \mu^m} \right) t^{-bm} \quad , \tag{4.8}$$

Putting these expression in (4.1) to (4.3), powers of t can be eliminated if the constants b & c are such that

$$2b+c-2=0, m\,b - c = 0 \quad , \tag{4.9}$$

$$\text{which give} \quad b= \frac{2}{m+2}, c= \frac{2m}{m+2} \quad . \tag{4.10}$$

The value b=1 corresponds to m=0 which show that the flow is uniform & homentropic . The approximated conditions for a strong shock are obtain by neglecting p_1 , so that (4.2) to (4.4) become

$$\rho_2 = \lambda^{-2} \rho_1 \quad , \tag{4.11}$$

$$p_2 = (1-\lambda^2) \rho_1 v^2 , \tag{4.12}$$

$$u = (1-\lambda^2) v \quad , \tag{4.13}$$

There is a condition to be satisfied by Π_2 to be a constant from(2.4)to(3.1) we get

$$\eta_2 = t\left(\frac{P}{c\rho_2^{\gamma}}\right)^{-1/\tau\alpha} = c^{+1/1\alpha}\, t^{(\tau\alpha+\tau c-2b-c-2)/\tau\alpha}\, \{P(\eta_2)\}^{-1/\tau\alpha}\{\Omega(\eta_2)\}^{1/\alpha}$$

by eliminating power of t we get $C + \alpha = 0$

which gives $\eta_2^{\tau\alpha} P(\eta_2) = C\{\Omega(\eta_2)\}^{\alpha}$

we also have

$$U(\eta_2) = (1-\ )^{2}\, b\, \mu = \frac{2b\mu}{\tau+1}, \qquad (4.14)$$

$$X(\eta_2) = \mu,$$

$$\frac{bX(\eta_1)}{U(\eta_2)} = \frac{1}{2}(\tau+1),$$

$$\Omega(\eta_2) = \lambda^{-2}\sigma u^{m} = \frac{\tau+1}{\tau-1}\sigma u^{n}, \qquad (4.15)$$

$$P(\eta_2) = \frac{2\sigma b^{2}\mu^{m+2}}{(\tau+1)^{2}}. \qquad (4.16)$$

Hence $$A^{2}(\eta_2) = \frac{2\tau(\tau-1)}{(\tau+1)^{2}}\, b^{2}\mu^{2}. \qquad (4.17)$$

Finally the equation to determine Π_2 become

$$\Pi_2^{\tau\alpha} = \frac{1}{2}\, c(\tau+1)^{\tau+1}\, \sigma^{\tau-1}\, b^{-2}\, \mu^{m(\tau-1)-2}\, (\tau-1)^{-\tau} . \tag{4.18}$$

So equations (4.14) to (4.18) constitute the boundary conditions for strong shock problem

Hence equation (3.18) with $\alpha+c=0$ and special values of the constant etc becomes

$$\frac{dY}{dZ} = \frac{Y}{Z}\,\frac{(m+2)-mY-2Z(n+1)}{mY(-\tau-1)+2(m+2)(1-Z)+(2nY)(1-\tau-2Z)/(\tau-1)} , \tag{4.19}$$

Boundary values for Y and Z are easily obtained from (4.14) & (3.11) and leads to simple value given as

$$Y_1 = Y(\Pi_2) = -\frac{2}{Y-1} , \quad Z_1 = Z(\Pi_2) = \frac{2Y}{\tau-1} . \tag{4.20}$$

Then the slope dY/dZ at this point is given by

$$\left(\frac{dY}{dz}\right)_{Y_1 Z_1} = \frac{(m-2)(\tau+1)-4nY}{4\tau(n+1)(\tau+1)} . \tag{4.21}$$

The values of Y,& Z , define unique integral curves .

5. C O N C L U S I O N

Altough a general description for any value of m is posible , we examine only for m=1 and 2 and making a comparative study of slopes and the different flows considered for m=1 , τ = 1.4 , we have

$$\left(\frac{dY}{dZ}\right)_{-5,7} = -0.1786 \quad . \tag{5.10}$$

Similarly for m=2 , τ=1.4 , we have

$$\left(\frac{dY}{dZ}\right)_{-5,7} = 0, \text{ when } n=0 \tag{5.20}$$

$$\left(\frac{dY}{dZ}\right)_{-5,7} = -0.2083 \text{ , when } n=1 \tag{5.30}$$

$$\left(\frac{dY}{dZ}\right)_{-5,7} = -0.2087 \text{ , when } n=2 \tag{5.40}$$

By observing the slopes in above cases at point $(Y_1 Z_1)$ it is seen that for m=1 , angle of slope has a decreasing tendency as the flow is takes from one dimensional to spherical and spherical to cylindrical where as for m=2 the tangent to integral curves is parallel to Z axis for one dimensional flow but flow truns through small angle and remain almost stationry in the case of sphericl and cylindrical flows .

REFERENES

(1) S.W. Yuan ;
An Analytical Approch to hyper velocity impact"AIAA" journale 2, (1964), 11667

(2) J.L White & S.W.Yuan ;
"Viscous effects on Hyper velocity impact JAP 42,(1979) , 4186

(3) J.L Whit & S.W yuan ;
"Influance of velocity on Blast wave, Solution as applied hgper velocity J.of Engineering Scince , 10 , (1982)

(4) W.J Rae and S.W Yuan ;
NSSA (1165) (R - 5425)

(5) Kynch J.L ;
Phys fluid 155, (1974), 4136

(6) Taylor G.J ;
Proc Roy Soc London 200 , (1950)

CHAPTER VII

ANALYTICAL SOLUTIONS OF CYLINDRICAL SHOCK WAVES IN A ROTATING GAS WITH AZIMUTHAL MAGNETIC FIELD

(1) INTRODUCTION

The propagation of hydromagnetic shock through a rotating gas being close to the actual situation of thunder. Kumar and Prakash [1,2,3] recently using C.C.W. [4,5,6] method have investigated the propagation of weak and strong diverging cylindrical hydromagnetic gas taking density distribution variable but axial component of magnetic field of constant strength.

In this chapter, using the C.C.W. method we have considerd the problems of the propagation of a converging and diverging cylindrical shock wave through a rotating gas under the influence of a magnetic field of constant axial and azimuthal components,simultaneously for both weak and strong waves.We have assumed initial distribution is constant.

The analytical expression for shock velocity and shock strength have been obtained for weak shock under the two conditions, namely when the magnetic field is weak and when the magnetic field is strong.For strong shock also we have considered two cases,i.e.when the magnetic field is strong and when ρ_0/ρ_1 is approximately equal to $\tau+1/\tau-1$, which is purely a non magnetic case.

(2) BASIC EQUATIONS BOUNDARY CONDITION AND AN ANALYTIC EXPRESSION FOR SHOCK VELOCITY

The equation governing the cylindrically symmetrical flow of rotating gas under the influence of transverse magnetic field are written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} + \frac{\mu}{2\rho}\frac{\partial}{\partial r}\left(H_\theta^2 + H_z^2\right) + \frac{\mu H_\theta^2}{r} = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(Vr) = 0 \quad ,$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0 \quad , \qquad (1)$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \quad ,$$

$$\frac{\partial H_\theta}{\partial t} + u\frac{\partial H_\theta}{\partial r} + H_\theta\frac{\partial u}{\partial r} = 0 \quad ,$$

$$\frac{\partial H_z}{\partial t} + \frac{\partial H_z}{\partial r} + H_z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0 \quad ,$$

where u , P and ρ are the velocity , pressure and density behind the shock front respectively while H_θ and H_z are azimuthal and axial components of magnetic field H .

The magnetogasdynamic shock conditions can be written in terms of a single parameter .

$$N = \frac{\rho_1}{\rho_0} \quad \text{as}$$

$$\rho_1 = \rho_0 N , \; H_1 = H_0 N , \; u_1 = \frac{N-1}{N} U ,$$

$$U^2 = \frac{2N}{(\tau+1) - (\tau-1) N} \left[a_0^2 + \frac{b_0^2}{2} + \{ (2-\tau) N + \tau \} \right] , \tag{2}$$

$$P_1 = P_0 + \frac{2 \rho_0 (N-1)}{(\tau+1) - (\tau-1) N} \left[a_0^2 + \frac{\tau-1}{4} b_0^2 (N-1)^2 \right] ,$$

where 0 and 1 denote respectively the states immediately ahead and behind the shock front ; U the shock veloicty, a the sound speed $(\tau P_0/\rho_0)$ and the Alfven speed is $(\mu H^2/\rho_0)$

where

$$H_0^2 = H_{\theta 0}^2 + H_{z0}^2 .$$

In the equilibrium state we have

$$\frac{1}{\rho_0} \frac{dp_0}{dr} + \frac{\mu}{\rho_0} \frac{H_\theta^2}{r} - \frac{V^2}{r} = 0 . \qquad (3)$$

Integrating the preceeding equation we get

$$P = K_1 \log \frac{A}{r} \text{ and } a_0 = K_2 \left[\operatorname{Log} \frac{A}{r} \right]^{1/2} , \qquad (4)$$

where $k_1 = (\mu H_{\theta 0}^2 + p' V^2)$, $K_2 = \left(\frac{Y k_1}{p'} \right)^{1/2}$ and

$[\mu H_{\theta o}^2 + \rho' V^2]$ Log A is constant of integration

WEAK SHOCKS

For a very weak shock we take the parameter N as

$$\frac{\rho}{\rho_0} = N = 1+\epsilon \quad , \epsilon << 1 \quad . \tag{5}$$

Now consider the two cases of weak and strong magnetic field

CASE - 1

When the magnetic field is weak i.e. $b_0^2 << a_0^2$ (i.e. $\mu H_\theta^2 << \tau P_0$)

and therefore the boundary conditions (2) reduce to

$$\rho_1 = \rho_0 (1+ \epsilon) \,, \; u_1 = b_0 \epsilon, \; H_{\theta 1} = H_{\theta 0} (1+ \epsilon)$$

$$H_{z_1} = H_{z_0} (1+\epsilon), P_1 = P_0 (1+\tau\epsilon), \tag{6}$$

$$U = (1+ \frac{\tau+1}{4}\epsilon) a_0 \quad .$$

CASE II

When magnetic field is strong $b_0^2 \gg a_0^2$ (i.e $\mu H_{\theta 0}^2 \gg \tau P_0$)

under these circumstances the boundary condition (2) reduce to

$$\rho_1 = \rho_0 (1 + \epsilon) \quad , u_1 = b_0 \quad , H_{\theta 1} = H_{\theta 0} (1 + \epsilon) \tag{7}$$

$$H_{z1} = H_{z0} (1 + \epsilon), \quad P_1 = P_0 (1 + \tau \epsilon) \quad ,$$

$$U = (1 + \frac{3}{4} \epsilon)\, b_0 \quad .$$

STRONG SHOCK

CASE - I

When the magnetic field is strong $b_0^2 \gg a_0^2$, (i.e., $\mu H_0^2 \gg \tau P_0$)

under this condition the boundary conditions (2) becomes

$$\rho_1 = \rho_0 N, H_1 = H_0 N, U_1 = \frac{N-1}{N} U, \quad \frac{P_1}{P_0} = \frac{X U^2}{a_0^2} + L, \tag{8}$$

where

$$X = \frac{\tau(\tau-1)(N-1)^3}{2N\{(2-\tau)N+\tau\}} \quad \text{and} \quad L = \frac{(\tau+1)N-(\tau-1)}{(\tau+1)-(\tau-1)N},$$

CASE - II

For strong shock when $b_0^2 \ll a_0^2$ (i.e. when $N \to [(\tau+1)/(\tau-1)]$ is small) the field may be regarded as independent of magnetic field. For converging shock the characteristic form of system of equations(2) is

$$dP + \mu(H_\theta dH_\theta + H_z dH_z) - \rho c du + \left[\frac{\rho c^2 u}{u-c} - \frac{\mu H_\theta^2 (u+c)}{u-c} + \frac{\rho c v^2}{u-c}\right] \frac{dr}{r} = 0, \tag{9}$$

where as the characteristic form for diverging shock is

$$dP+\mu(H_\theta \, dH_\theta + H_z \, dH_z) + \rho c\,du + \left[\frac{\rho c^2 u}{u+c} - \frac{\mu H_\theta^2 (u-c)}{u+c} - \frac{\rho c v^2}{u+c} \right] \frac{dr}{r} = 0 , \quad (10)$$

where $C^2 = a^2 + b^2 = \dfrac{\tau p}{\rho} + \dfrac{\mu(H_\theta^2 + H_z^2)}{\rho}$,

and $H_o^2 = H_\theta^2 + H_z^2$.

WEAK SHOCK WITH WEAK MAGNETIC FIELD

Substituting shock condition (6) into (9) and neglecting the second and higher order term of ϵ , since $\epsilon << 1$ we get

$$\left\{\frac{\mu}{\tau P_o} (H_{\theta o}^2 + H_{zo}^2) \right\} d\epsilon + \left\{ \frac{dP_o}{P_o} - \frac{da_o}{a_o} - \frac{dr}{r} + \frac{2\mu H_{\theta o}^2}{\tau P_o} - \frac{2dP_o}{\tau P_o} \right\} \epsilon = 0 . \quad (11)$$

Now substituting $\dfrac{dP_o}{P_o}$, $\dfrac{da_o}{a_o}$, $\dfrac{\mu H_{\theta o}^2}{\tau P_o}$ and $\dfrac{\mu H_{zo}^2}{\tau P_o}$,

in equation (11) we get

$$\frac{d\epsilon}{\epsilon} = \left[\frac{[\tau(1-4\beta_2^2)-4]}{\tau\beta_1^2} + \frac{1}{\beta_1^2} \log \frac{A}{r} \right] \frac{dr}{r} , \qquad (12)$$

where $$\beta_1^2 = \frac{\mu(H_{\theta o}^2 + H_{zo}^2)}{\tau K_1} ,$$

$$\beta_2^2 = \frac{\mu H_{\theta o}^2}{\tau K_1} .$$

Integration of equation (12) yields ,

$$\epsilon = \bar{K}_3 \, r^{[\tau(1-A\beta_2^2)-4]/\tau\beta_1^2} \exp \left\{ - \frac{1}{2\beta_1^2} \left(\log \frac{A}{r}\right)^2 \right\} , \qquad (13a)$$

where $\bar{K}_3$ is constant of integration .

In the case of diverging weak shock with weak magnetic field, we have the relation

$$\epsilon = K_3 r^{-1/2} \left(\log \frac{A}{r}\right)^{-q_1} \exp\left\{-\frac{3}{8}\beta_1^2 \left(\log \frac{A}{r}\right)^{-1}\right\}, \qquad (13b)$$

where K_3 is constant of integration and

$$q_1 = \left(\frac{3}{4} + \frac{\beta_1^2}{4}\right),$$

hence from equation (13a) and (13b) we have

$$\frac{u}{a_0} = 1 + \frac{\tau+1}{4} \bar{K}_3 r^{[\tau(1-4\beta_1^2)-4]/\tau\beta_1^2} \exp\left\{-\frac{1}{2\beta_1^2}\left(\log \frac{A}{r}\right)^2\right\}, \qquad (14)$$

$$\frac{u}{a_0} = 1 + \frac{\tau+1}{4} K_3 r^{-1/2} \left(\log \frac{A}{r}\right)^{-q_1} \exp\left\{-\frac{3}{8}\beta_1^2 \left(\log \frac{A}{r}\right)^{-1}\right\}, \qquad (15)$$

for converging and diverging shock respectively

WEAK SHOCK WITH STRONG MAGNETIC FIELD

Substituting the shock conditions (7) in (9) we get

$$d\epsilon+\{\frac{dP_o}{P_o} - \frac{\mu H_o^2}{\tau P_o}\left(\frac{dr}{r} + \frac{db_o}{b_o}\right)+ \frac{2\mu H_{\theta o}^2}{\tau P_o}\frac{dr}{r} - \frac{2dP_o}{\tau P_o}\} =0 \quad , \qquad (16)$$

substiuting the respective quantities in equation (16)

we get

$$\frac{d\epsilon}{\epsilon} =\left[\frac{\tau(1+\beta_1^2+2\beta_2^2 - 2)}{\tau}\right] \left(\log \frac{A}{r}\right)^{-1} \frac{dr}{r}, \qquad (17)$$

integrating equation (17) , we get

$$\epsilon =\bar{K}_4 \left(\log \frac{A}{r}\right)^{-q_2} , \qquad (18a)$$

where $\bar{K}_4$ is constant of integration and

$$q_2 = \frac{[\tau(1+\beta_1^2+2\beta_2^2) - 2]}{\tau} ,$$

when we consider diverging shock with strong magnetic field we get,

$$\epsilon =K_4 \left(\log\left(\frac{A}{r}\right)\right)^{(\beta_1^2-1)} \exp \{-2\beta_1^2 (1-\beta_1^2)\left(\log \frac{A}{r}\right)^{-1} , \qquad (18b)$$

where K_4 is constant of integration, equation (18a) and (18b) disclose that

$$\frac{u}{b_0} = 1 + \frac{3}{4}\bar{K}_4 \left(\log \frac{A}{r}\right)^{-q_2}, \tag{19}$$

and

$$\frac{u}{b_0} = 1 + \frac{3}{4}K_4 \left(\log \frac{A}{r}\right)^{\beta_1^2 - 1} \exp\left\{-2\beta_1^2 (1-\beta_1^2)\left(\log \frac{A}{r}\right)^{-1}\right\}, \tag{20}$$

for converging and diverging shocks respectively .

STRONG SHOCK

Substiuting shock condition (8) into (9),we get

$$dU \left[\frac{2x}{\gamma} - \frac{N-1}{2}\left(\frac{x}{N}\right)^{1/2}\right] + U\left[-\frac{2x}{\gamma}\frac{dp_0}{p_0} - \frac{2x}{\gamma}\frac{da_0}{a_0} + \frac{x(N-1)}{(N-1)-\sqrt{Nx}}\frac{dr}{r}\right]$$

$$+\left[L\frac{dp_0}{\rho_0} - \frac{\tau K N^2 \beta_1^2}{\rho_0}\left\{\frac{(N-1)+\sqrt{Nx}}{(N-1)-\sqrt{Nx}}\right\}\frac{dr}{r} + \frac{N\sqrt{Nx}}{[(N-1)-\sqrt{Nx}]}\right] = 0, \tag{21}$$

Substituting the respective values , we get

$$\frac{dU^2}{dr} + \frac{B}{r_1} U^2 - C r^{-1} = 0 ,$$

where $\quad B = \bar{B} / M \quad , \quad C = \bar{C} / M$

$$\bar{B} = \frac{x\,(N-1)}{(N-1)\,\sqrt{xN}} \quad , \; M = \frac{x}{Y} - \frac{(N-1)}{2} \left(\frac{x}{N}\right)^{1/2} ,$$

$$\text{and } \bar{C} = \frac{L K_1}{\rho'} + \frac{\tau K_1 N^2 \beta_1^2}{\rho'} \left\{ \frac{(N-1) + \sqrt{Nx}}{(N-1) - \sqrt{Nx}} \right\} - \frac{N \sqrt{Nx}}{(N-1) - \sqrt{Nx}}$$

integration of (22) yields ,

$$U^2 = \bar{K}_5 \; r^{-B} + \frac{C}{B}$$

where $\bar{K}_5$ is constant of integration

In the case of diverging strong shock with strong magnetic filed we have ,

$$U^2 = K_5 \; r^{-B} + \frac{C}{B}, \tag{24}$$

where K_5 is constant of integration . Finally , for converging and diverging shocks we have

$$\frac{U}{a_o^2} = \frac{1}{K_2^2 \, \mathrm{Log}\, \frac{A}{r}} \left(\bar{K}_5 \, r^{-B} + \frac{C}{B} \right) , \tag{25}$$

$$\frac{U}{a_o^2} = \frac{1}{K^2 \, \mathrm{Log}\, \frac{A}{r}} \left(K_5 \, r^{-B} + \frac{C}{B} \right) \tag{26}$$

respectively .

In the last, the expressions for the velocity, the density, and the pressure of the gas just behind the shock surface are.

(i) <u>**Weak shock with weak Magnetic Field .**</u>

$$u = k_2 \bar{k}_3 \left(\log \frac{A}{r}\right)^{\frac{1}{2}} r^{[\tau(1-4\beta_2^2)-4)/\tau\beta_1^2} \exp\left\{- \frac{1}{2\beta_1^2} \left(\log \frac{A}{r}\right)^2 \right\}, \tag{27a}$$

$$u = k_2 k_3 r^{-\frac{1}{2}} \left(\log \frac{A}{r}\right)^{(1-2q_1)} \exp\left\{- \frac{3}{8} \beta_1^2 \left(\log \frac{A}{r}\right)^{-1} \right\} , \tag{27b}$$

$$\rho = \rho' [1+\bar{k}_3 r^{[\tau(1-4\beta_2^2)-4]/\tau\beta_1^2} \exp\left\{- \frac{1}{2\beta_1^2} \left(\log \frac{A}{r}\right)^2 \right\} , \tag{28a}$$

$$\rho = \rho' [1+k_3 r^{-\frac{1}{2}} \left(\log \frac{A}{r}\right)^{-q_1} \exp\left\{- \frac{3}{8} \beta_1^2 \left(\log \frac{A}{r}\right)^{-1} \right\}, \tag{28b}$$

$$P = K_1 [\log \frac{A}{r} + \tau \bar{k}_3 r^{[\tau(1-4\beta_2^2)-4]/\tau\beta_1^2} (\log \frac{A}{r}) \exp\{- \frac{1}{2\beta_1^2} (\log \frac{A}{r})^2 \} \tag{29a}$$

$$P = K_1 [\log \frac{A}{r} + \tau k_3 r^{-1/2} (\log \frac{A}{r})^{(1-q_1)} \exp \{- \frac{3}{8} \beta_1^2 (\log \frac{A}{r})^{-1} \}. \tag{29b}$$

(2) <u>Weak Shock With Strong Magnetic Field</u>

$$u = \beta_1 K_2 \bar{K}_4 (\log \frac{A}{r})^{-q_1} , \tag{30a}$$

$$u = \beta_1 K_2 \bar{K}_4 (\log \frac{A}{r})^{\beta_1^2 - 1} \exp \{-2\beta_1^2 (1-\beta_1^2) (\log \frac{A}{r})^{-1} \}, \tag{30b}$$

$$\rho = \rho' [1+\bar{K}_4 (\log \frac{A}{r})^{-q_2}] , \tag{31a}$$

$$\rho = \rho' [1+\bar{K}_4 (\log \frac{A}{r})^{(\beta_1^2 -1)} \exp \{-2\beta_1^2 (1-\beta_1^2) (\log \frac{A}{r})^{-1} \}], \tag{31b}$$

$$P = K_1 [\log \frac{A}{r} + \tau \bar{K}_4 (\log \frac{A}{r})^{(1-q_2)}] , \tag{32a}$$

$$P = K_1 \left[\log \frac{A}{r} + \tau K_4 \left(\log \frac{A}{r}\right)^{\beta_1^2} \exp\left\{-2\beta_1^2 (1-\beta_1^2) \left(\log \frac{A}{r}\right)^{-1}\right\}\right], \tag{32b}$$

(3) Strong Shock With Strong Magnetic Field

$$u = \frac{(N-1)}{N} \left[\bar{K}_5 r^{-B} + \frac{C}{B}\right]^{1/2}, \tag{33a}$$

$$u = \frac{(N-1)}{N} \left[K_5 r^{-B} + \frac{C}{B}\right]^{1/2}, \tag{33b}$$

$\rho = \rho' N$. For converging & diverging cases

$$P = K_1 \log \frac{A}{r} \left[\frac{X(\bar{K}_5 r^{-B} + \frac{C}{B})}{K_2 \left(\log \frac{A}{r}\right)^{1/2}} + L\right], \tag{34a}$$

$$P = K_1 \log \frac{A}{r} \left[\frac{X(K_5 r^{-B} + \frac{C}{B})}{K \left(\log \frac{A}{\gamma}\right)^{1/2}} + L\right], \tag{34b}$$

where sections (a) and (b) denote the expression for converging and diverging shocks respectively .

REFERENCES

(1) Kumar ,S, and Prakash,R:

Proc.Indian Soc Theor.Appl Mech. 27(2),11, (1982)

(2) Kumar,S; and Prakash,R:

IL NUOVO Cimento, Vol 77B,N.2,191-201 , (1983)

(3) Kumar,S.Saxena, A. K. and Prakash,R:

Nuovo Cimento 7D , (1986)

(4) Chester W:

Phil Mag 45(7),1293 , (1954)

(5) Chisnell,R.F.:

Proc. Roy,Soc, London Ser.A232, 350 ,(1955)

(6) Whitham G.B:

J Fluid Mech. 4.337 ,(1958)

(7) Singh ,J.B: & Pandey ,. K.s.:

Astrophysics and Space Science 148, 85-93 ,(1988)

CHAPTER VIII

EFFECT OF ARTIFICIAL VISCOSITY ON THE EXPANSION OF DISCONTINUITIES IN A ROTATING INTERPLANETRY MEDIUM

(1) INTRODUCTION

The propagation of strong shock waves in space, due to a surface explosion or impact has been treated in many levels of approximation. In one of these, an attempt is made to account for the material strength by including Newtonian viscosity term. S.W.Yuan[1,2,3] approximated the solutions of equations by seeking quasi- similar solution. In all of these solution the viscosity coefficient is taken to be at most a function of time but independent of space coordiantes. Pai [4,5] and Kumar [6] discussed the propagation of hydromagnetic cylindrical shock through a self - gravitating gas showing its velocity only for strong shocks. Recently Singh and Mishra [7] obtained analytical relations for shock velocity and shock strength and the expressions for the pressure, the density and the particle velocity immediatly behind the shock assuming the fact that the initial density and azimuthal magnetic field distributions variables.

In the present chapter the characteristic method (Chester [8] Witham [9]) is applied to abtain expressions of the density, the pressure, the particle velocity just behind the shock propagating in a rotating atmosphere. The effect of coriolis force is taken into account. Since the velocity effect has tendency to smoothen out such discontinuities, the artfical viscosity coefficient suggested by Ritchmyer Von Neumann [10]

is introduced. The problem is discussed for two different cases (i) for weak shocks and (ii) for strong shocks respectively .

(2) BASIC EQUATION BOUNDARY CONDITIONS AND ANALYTICAL EXPRESSION FOR SHOCK VELOCITY

The equations governing the cylindrically- Symmetric flow of gas undar the influence of its own coriolis forces in the presence of transverse magnetic field,Following Witham [7] are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}(p+q) + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} - \frac{V^2}{r} = 0 ,$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \rho\frac{u}{r} = 0 , \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - \frac{[\gamma p + (\gamma-1)q]}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 ,$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} = 0 ,$$

$$\frac{\partial}{\partial t}(vr) + u\frac{\partial}{\partial r}(vr) = 0 ,$$

$$q = \frac{1}{2} \rho r^2 K^2 \frac{\partial u}{\partial r} \left\{ \left| \frac{\partial u}{\partial r} \right| - \left(\frac{\partial u}{\partial r} \right) \right\} ,$$

where u (r,t), p(r,t), H(r,t) vr(r,t), and $\rho(r,t)$ respectively represent the velocity , pressure , magnetic field at distance (r)and time (t) and the density where $a^2 = \tau p/\rho$. and "q" is the artificial viscosity.

The magnetic hydrodyanamic condition can be written in terms of a single parameter $N = \rho_1 / \rho_0$ as

$$\rho_1 = N\rho_o , H_1 = NH_o , u_1 = (1 - \frac{1}{N}) U ,$$

$$U^2 = \frac{2N}{(\tau+1) - (\tau-1)} [a_o^2 + \frac{b_o^2}{2} (2 - \tau) N +\tau)] , \qquad (2)$$

$$p_1 = p_o + \frac{2 \rho_0 (N-1)}{(\tau+1)- (\tau-1)N} [a_o^2 + \frac{\tau-1}{4} b_o^2 (N-1)^2] ,$$

where o and 1 respectively stand for the states just ahead and just behind the shock front ; U is the shock velocity,a_o is the sound speed given by $(\tau p / \rho_o)^{1/2}$, and b_o is the Alfven speed given by $\left(\frac{\mu H_o^2}{\rho_o}\right)^{1/2}$, where μ is magnetic permeability .

(2.1) WEAK SHOCK

For very weak shock we take the parameter as

$$\rho_1 / \rho_0 = 1+ \epsilon \quad . \qquad (3)$$

Now considere the two cases of weak and strong magnetic field .

CASE I

For weak magnetic field $b_0^2/a_0^2 \ll 1$. Under this condition the boundary conditions (2) for very weak shock reduce to

$$\rho_1 = \rho_o (1+\epsilon) , \; H_1 = H_o (1+ \epsilon), \; u_1 = \epsilon a_o , \; P_1 = P_o (1 + \tau \epsilon) , \qquad (4)$$

$$U = (1+ \frac{\tau+1}{4} \epsilon) a_o \quad .$$

CASE II

For storng boundary magnetic field $b_o^2 / a_o^2 \gg 1$, using this condition and equation (3) the boundary conditions (2) reduce to

$$\rho_1 = \rho_o (1+ \epsilon), \; H_1 = (1+ \epsilon) H_o , \; u_1 = \epsilon b_o ,$$

$$P_1 = (1+\tau\epsilon)\, P_o \ , \ U = (1+ \frac{3}{4} \epsilon\,)\, b_o \ .$$

(2.2) STRONG SHOCK

In the limiting case of a strong shock $\frac{\rho_1}{\rho_0}$ is large; and in the presence of the magnetic case thie can be brought about in two ways.

CASE I

The purely nonmagnetic way when $N \rightarrow \frac{\tau+1}{\tau-1}$ is small .

When $b_o^2 >> a_o^2$ or when $\mu H_o^2 >> P_o$ that is when the ambient magnetic pressure is large compared with the ambient fluid pressure. In terms of N, the boundary conditions (2), now become

$$\rho_1 = N\rho_0 \ , \ H_1 = NH_0 \ , \ u = (1- 1/N\,)\, U, \frac{p_1}{p_o} = X(N)\, \frac{U^2}{a_o^2} + Y(N), \quad (6)$$

where

$$X(N) = \frac{(\tau-1)\ (N-1)^3}{2N[(2-\tau)\ N + \tau]} \quad \text{and} \quad Y(N) = \frac{N\ (\tau+1) - (\tau-1)}{(\tau+1) - (\tau-1)\ N} \ .$$

For diverging shock the characteristic form of system of equation (1), that is the form in which equation contains derivatives only in (r,t) plane is

$$dP+\rho c\,du+H\,dH-H^{2}\frac{(u-c)}{(u+c)}\frac{dr}{r}+\frac{\rho c^{2}u}{(u+c)}\cdot\frac{dr}{r}-\frac{\rho c^{2}v^{2}}{(u+c)}\frac{dr}{r}$$

$$+\frac{c q_{r}}{(u+c)}+\frac{q(\tau-1)dr}{(u+c)}+\frac{q(\tau-1)u}{(u+c)}\frac{dr}{r}=0\quad,\qquad(7)$$

where $c^{2}=a^{2}+b^{2}=\frac{\tau p}{\rho}+\frac{H^{2}}{\rho}$, $\mu=1$

Now we substitute the shock condition 4 or 5 or 6 into relation (7) .A first order differential equation in (r) or u is obtained which gives the shock . For weak shock in the presence of weak transverse magnetic field , on substituting the shock condition (4) in (7), neglected the second and heigher terms of ϵ (ϵ <1) , we obtain

$$\left[2+\frac{H_{o}^{2}}{P_{o}}\right]d\epsilon+\left[\frac{dp_{o}}{P_{o}}+\frac{dH_{o}^{2}}{\tau P_{o}}+\frac{dr}{r}+\frac{da_{o}}{a_{o}}+\frac{q_{r}}{\tau P_{o}}dr+\frac{q(\tau-1)dr}{\tau P_{o}\,r}\right]\epsilon$$

$$= -a_o^{-2}\left[\frac{dP_o}{\rho_o} + \frac{1}{2}\frac{dH_o^2}{\rho_o} + \frac{H_o^2}{\rho_o}\frac{dr}{r} - V^2\frac{dr}{r} - \frac{q_r}{\rho_o}\,dr\right], \qquad (8)$$

But by hydrostatic equilibrium prevalent in front of the shock

$$V^2\frac{dr}{r} = \frac{dP_o}{\rho_o} + \frac{1}{2}\frac{dH_o^2}{\rho_o} + \frac{H_o^2}{\rho_o}\cdot\frac{dr}{r} - \frac{q_r}{\rho_o}\,dr. \qquad (9)$$

Hence eqution (8) reduce to

$$\frac{d\epsilon}{\epsilon} = -\frac{1}{2}\left(1-\frac{H_o}{2\tau p_o}\right)\left(\frac{dp_o}{p_o} + \frac{dH_o}{\tau p_o} + \frac{dr}{r} + \frac{da_o}{a} - \frac{q_r\,dr}{\tau p_o} + \frac{q(\tau-1)}{\tau p_o}\frac{dr}{r}\right) \qquad (10)$$

Also we have

$$P_o = -\left[\frac{p r^{-w} V^2}{w} + \frac{(\bar{w}-1)}{2\bar{w}} H_c^2 r^{-2\bar{w}}\right].$$

Incerting $\bar{w} = w/2$, we get

$$P_o = \left[\frac{p_c V^2}{w} - \frac{H_c^2}{2w}\right] r^{-w},$$

$$P_o = K_1 r^{-w} \quad , \tag{12}$$

$$\rho_o = \rho_c r^{-w} \quad , \tag{13}$$

$$a_o = K_2 \quad ,$$

$$\text{where} \quad K_1 = \left[\frac{\rho_c V^2}{w} - \frac{H_c^2}{2w} \right] \quad ,$$

and

$$K_2 = \frac{\tau K_1}{p_c} \quad ,$$

Positively and finiteness of the equation pressure as defind by equation (12) requires that the constant (w) obey the inequality .

$$1<W<2 \quad , \tag{14}$$

Substituting dp_o / p_o and da_o / a_o in equation (10) we get

$$\frac{d\epsilon}{\epsilon} = \frac{1}{4} \left[2(w-1) + \beta^2 (w+1)\right] \frac{dr}{r} + \frac{1}{4}(2-\beta^2) \frac{1}{\tau K_1}\left[r^w dq - r^{(w-1)} q(\tau-1)dr\right] \quad . \tag{15}$$

Integrating the above equation we get, where for whole region $\int \partial q = 0$,

$$\epsilon = r^{\frac{1}{2}[2(w-1)+\beta^2(w+1)]} e^{-\frac{1}{2}(2-\beta^2)\, r^w (\tau-1)\, q/\tau K_1 w}, \quad (16)$$

Where $\beta^2 = H_c^2/\tau K_1$ and K is constant of integration. With the aid of equation (4), we can wright .

$$\frac{U}{K_2} = [1+\frac{\tau+1}{4} K r^{\frac{1}{2}(2(w-1)+\beta^2(w+1)} e^{-\frac{1}{2}(2-\beta^2)(\tau-1)\, r^w q/\tau k_1 w}, \quad (17)$$

$$\frac{U}{a_o} = [1+\frac{\tau+1}{4} K r^{\frac{1}{2}(2(w-1)+\beta^2(w+1)} e^{-\frac{1}{2}(2-\beta^2)(\tau-1)\, r^w q/k_1 w\tau}, \quad (18)$$

Similarly for a strong magnetic field , by use of the shock condition (5) in (7) we get

$$\frac{d\epsilon}{\epsilon} = -\frac{1}{2}(1-\frac{\gamma P_o}{2H_o^2})\, [\frac{\tau}{H_o^2} dP_o + 2\frac{dH_o}{H_o} + \frac{db_o}{b_o} + \frac{dr}{r} - \frac{q\,\frac{dr}{r}}{H_o^2}$$

$$+ \frac{(\tau-1)}{H_o^2} \frac{dr}{r}] \quad , \qquad (19)$$

Substituting the value of dP_o , dH_o / H_o and db_o / b_o in the equation (19) and integrating we get

$$E = \bar{k}\, r^{-\frac{1}{4}[\frac{(3w-1)}{\beta^2} + 2(1-w)]} \cdot e^{-\frac{1}{2}(1-\frac{1}{2}\beta^2)\, q r^w (\tau-1)/w k_1 \beta^2} , \qquad (20)$$

where $\bar{k}$ is constant of integration. With the equation of (5) we write

$$\frac{U}{K_2} = [1 + \frac{3}{4} \bar{k} r^{-\frac{1}{4}[\frac{(3w-1)}{\beta^2} + 2(1-w)]} \cdot e^{-\frac{1}{2}(1-\frac{1}{2}\beta^2) q r^w (\tau-1)/w k_1 \beta^2 \tau} \qquad (21)$$

$$\frac{U}{a_2} = [1 + \frac{3}{4} K r^{-\frac{1}{4}[\frac{(3w-1)}{\beta^2} + 2(1-w)]} \cdot e^{-\frac{1}{2}(1-\frac{1}{2}\beta^2) q r^w (\tau-1)/w k_1 \tau \beta^2 \, 2} \qquad (22)$$

For strong shock, if we substitute the shock condition (6) in (7) we get -

$$[\frac{x}{\tau} + \frac{1}{2}(N-1)(\frac{x}{N})^{\frac{1}{2}}]^2 dU + [\frac{x}{\tau} \frac{dP_o}{P_o} - 2 \frac{x}{\tau} \frac{da}{a_o} + \frac{(N-1)}{(N-1)+(xN)} \frac{1}{2} \frac{dr}{r}] U^2$$

$$+\Big[Y\frac{dP_o}{\rho_o}+\frac{N^2H_o\,dH_o}{\rho_o}-\frac{H_oN^2[(N-1)-(XN)^{1/2}]}{\rho_o[(N-1)+(XN)^{1/2}]}\frac{dr}{r}-\frac{N(NX)^{1/2}v^2}{[(N-1)+(XN)^{1/2}]}\frac{dr}{r}$$

$$+\frac{(XN)^{1/2}q\,dr}{\rho_o[N-1]+(XN)^{1/2}}+\frac{q(\tau-1)(N-1)}{\rho_o[(N-1)+(XN)^{1/2}]}\frac{dr}{r}\Big]=0\ , \quad (23)$$

substituting the values of $\frac{da_o}{a_o}$ and $\frac{dp_o}{p_o}$ we find that

$$\frac{dU^2}{dr}+B\frac{U^2}{r}+\frac{C}{r}+Er^{w}q_r+Dr^{w-1}q=0\ ,$$

where $q_r = q/r$,

and $\bar{B}=AB$, $\bar{C}=AC$, $\bar{D}=AD$, $\bar{E}=AE$,

$$\bar{B}=\Big[\frac{(N-1)\tau}{(N-1)+(XN)}{}^{1/2}-W\Big]\frac{x}{\tau}\ ;\ A=\frac{x}{\tau}+\frac{1}{2}(N-1)\left(\frac{x}{N}\right)^{1/2}\ ,$$

$$\bar{C} = \left[\frac{-\gamma K_1}{p_c} - \frac{N^2 H_c^2}{p_c} \; \frac{[(N-1)-(XN)^{1/2}]}{[(N-1)+(XN)^{1/2}]} - \frac{W}{2} \; \frac{N^2 H_c^2}{p_c} \right.$$

$$\left. - \frac{N(NX)^{1/2} - V^2}{[(N-1)+(XN)^{1/2}]} \right] ,$$

$$\bar{D} = \frac{(\gamma-1)\;(N-1)}{p_c\,[N-1] + (XN)^{1/2}]} , \quad \bar{E} = \frac{(XN)^{1/2}}{p_c\,[(N-1) + (XN)^{1/2}]} .$$

On integration , equation (24) gives

$$U^2 = \bar{\bar{K}}\, r^{-B} - \left(\frac{C}{B} + \frac{r^W D\, q}{W + B} \right) ,$$

where $\bar{\bar{K}}$ is constant of integration and we assumed that for whole region $\int q_r \, dr = 0$.

(3) DISCUSSION

Finally the expression for the pressure and the particle velocity just behind the shock for above can be expressed as

$$P= K_1 r^{-w} + K K_1 r^{\frac{1}{4}[-2(w+1)+\beta^2 (w+1)]-\frac{1}{4}[(2-\beta^2)(\tau-1)] e^{qr^w/\tau k_1 w}}$$

$$\rho=\rho_c r^{-w} + K\rho_c r^{\frac{1}{4}[-2(w+1)+\beta^2 (w+1)]-\frac{1}{4}(2-\beta^2) r^w e^{q(\tau-1)/\tau K_1 w}}$$

$$u = \epsilon a_o = KK_2 r^{\frac{1}{4}[2(w-1)+\beta^2 (w+1)]-\frac{1}{4}(2-\beta^2) r^w e^{q(\tau-1)/\tau K_1 w}} \tag{27}$$

$$P= K_1 r^{-w} + \bar{K}K_1 r^{\frac{1}{4}[(3w-1)/\beta^2+2(1-3w)]-\frac{1}{2}[1-\frac{1}{2}\beta^2] e^{qr^{-w}(\tau-1)/\tau wK_1 \beta^2}}$$

$$\rho= \rho_c r^{-w} + \rho\bar{K}_c r^{\frac{1}{4}[(3w-1)/\beta^2+2(1-3w)]-\frac{1}{2}(1-(\frac{1}{2}\beta^2)) e^{qr^w(\tau-1)/\tau wk_1 \beta^2}}$$

$$u= \bar{K}K_2 r^{\frac{1}{4}[2(w-1)+\beta^2 (w+1)]-\frac{1}{2}(1-(\frac{1}{2}\beta^2)) e^{qr^w(\tau-1)/\tau wk_1 \beta^2}}, \tag{28}$$

and for strong shock

$$P = \frac{K}{K_2}[x(N)Kr^{-(w+B)} -(C/B)r^{-w} - (\frac{dq}{B+w}) +K r^{-w} Y(N), \tag{29}$$

$$u = (1-(1/N))\ (K\, r^{-B} - \frac{C}{B} - D^{*} q r^{[w/(W+B)]^{1/2}}$$

The expression (17) and (21) represent the propagation of weak diverging cylindrical shock wave through a rotating gas with effect of artificial viscosity in the presence of transverse weak and strong magnetic field respectively .

In relation (25) first term represents the solution obtained by the Witham, showing the effect of density distribution . The second term arises obeviously on account of the coriolis force and the magnetic field . The velocity of magnetogasdynamic shock wave is determined by first term only when magnetic field is strong , for large values of r , the first trem is very small in comparison to the second term .

As a consequence, the shock velocity attains a minimum value for a certain propagation distance beyond which the coriolis trem stand dominating and shock propagation distance corresponding to the minimum value of the shock velocity may be obtained by equating

$$\frac{du}{dr} = 0 \quad .$$

REFERENCES

(1) S.W.Yuan and C . N . Scully ;

A new approch to hyper velocity impact theory, PP 599-615 of the procedings of the 9th annual meeting of the American astronoutical Society (1963)

(2) S.W.Yuan ;

An analytical approch to hyper velocity impact ; AIAA Journale (1971)

(3) S.W.Yuan and J.L. Whitesider

Viscouss effects on hyper velocity impact , Journale of applied Phy . ; 42 , 4158 , (1971)

(4) S.I.Pai;

Introduction to the theory of compressible flow , D Von Nostrand conpany INC, New york (1959)

(5) S.I.Pai;

Magnetogasdynamic amd plasma dynamics springer - verlag (1962)

(6) S.Kumar and R. Prakash ;

IL Nuovo Cimento vol 77B,N2 PP 191 - 201, (1983)

(7) J.B.Singh and Mishra ;

The mathematical society Banaras University vol (3) , (1987)

(8) W.Chester ;

Phil Mag (7) V45, PP, 1223 - 301 , (1954)

(9) G.B Witham ;

J.Fluid Mech V 4, PP 337 - 60 , (1958)

(10) J. Von Newmann and R.D Richtmger

J. appl . phys; 21, 232 (1950)